

Practical nonlinear shear rheometry of polymers

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Useful reading

1. Macosko, Rheology: Principles, measurements and applications (1994)
2. Doi, Edwards, The theory of polymer dynamics, Oxford (1986)
3. Ferry, Viscoelastic Properties of Polymers (1980)
4. Larson, Structure and rheology of complex fluids (1999)
5. Rubinstein, Colby, Polymer Physics (2003)
6. Graessley, Polymer liquids and networks: Structure and rheology (2008)
7. Dealy, Read and Larson, Structure and rheology of molten polymers, 2nd ed. (2018)
8. Wang, Nonlinear polymer rheology, Wiley, 2018
9. Dealy, Wissbrun, Melt rheology and its role in plastics processing

Syllabus

Linear versus nonlinear behavior of polymers.

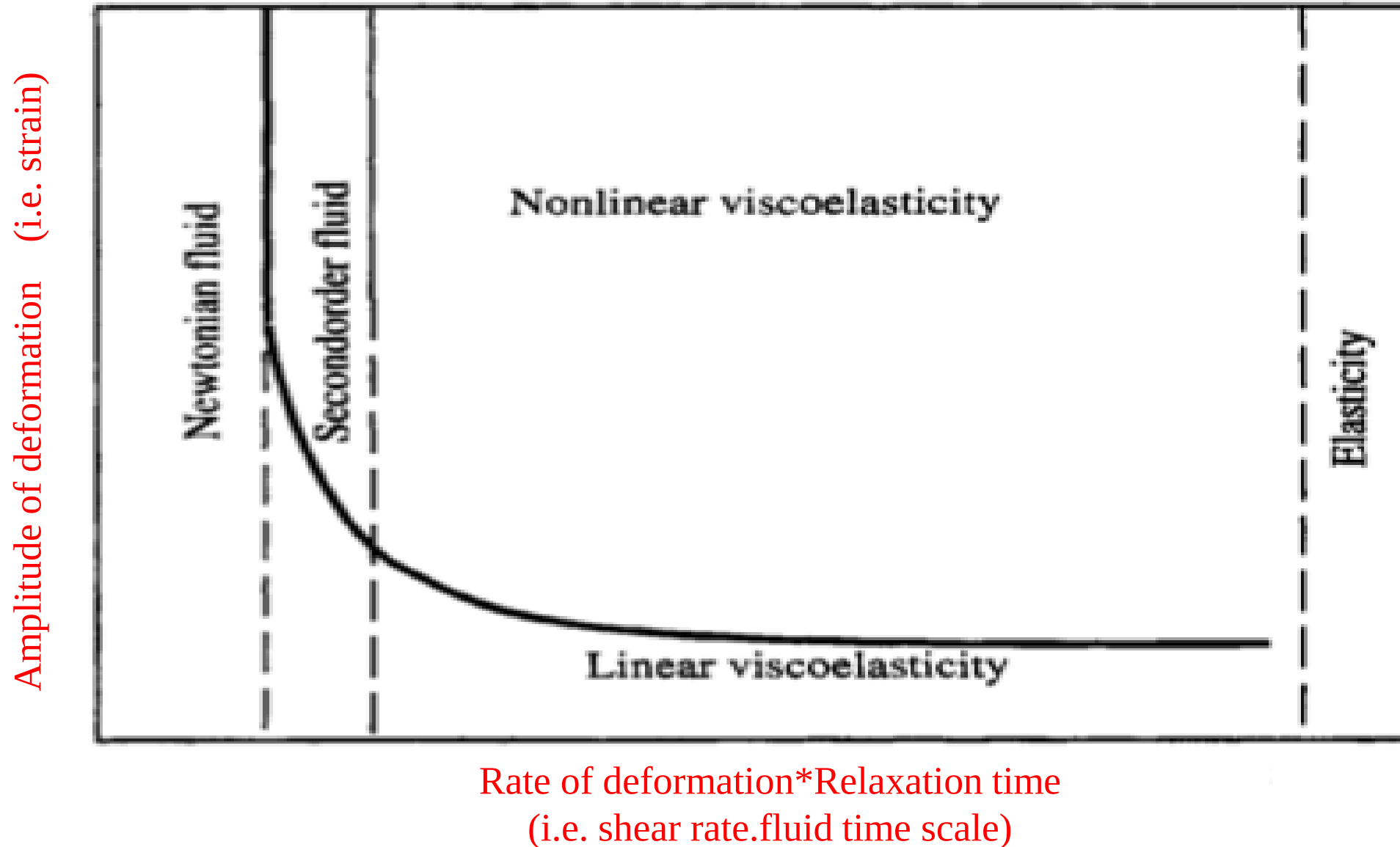
Shear thinning

How to obtain data in simple shear flow

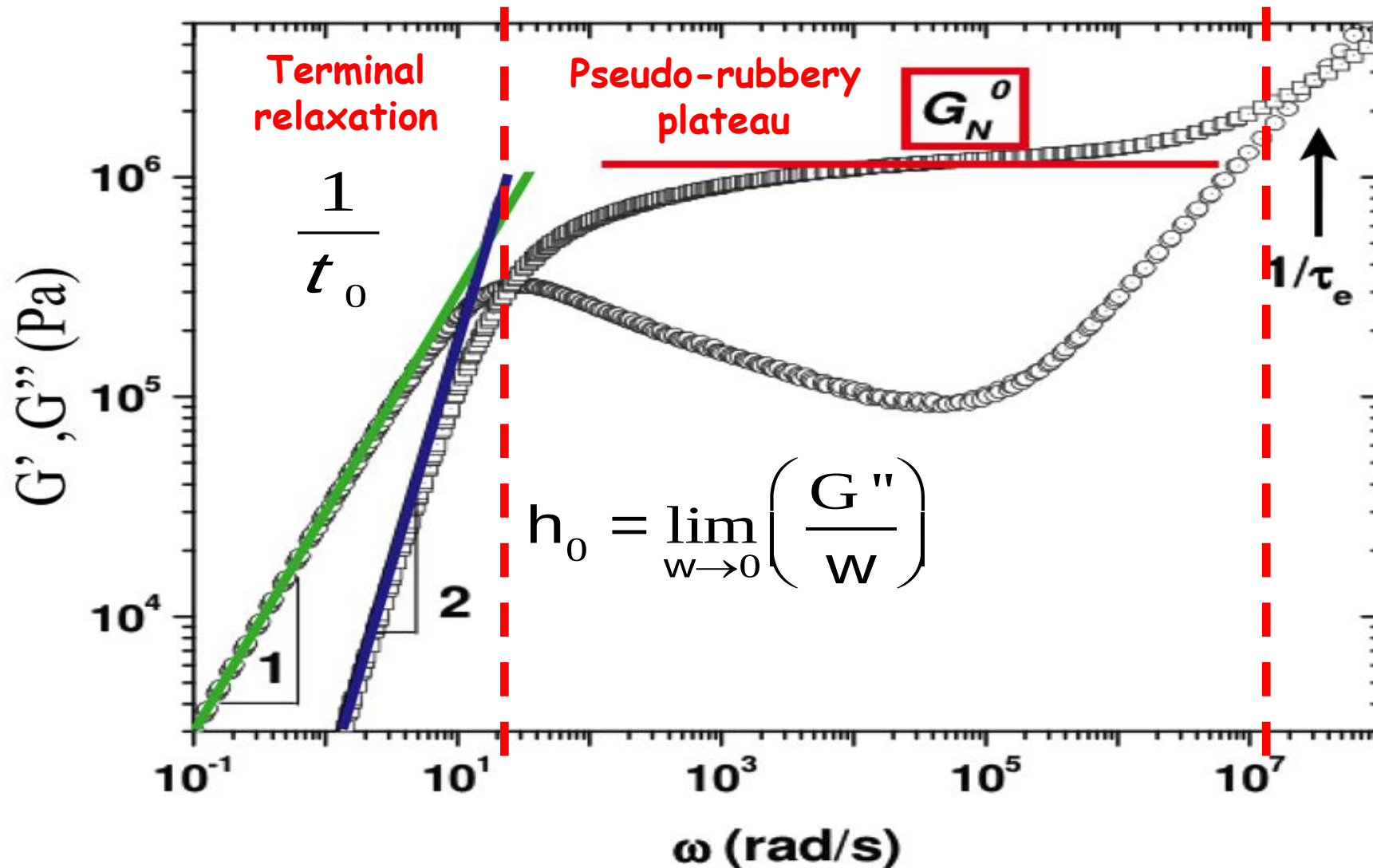
Polymers in simple steady shear flow.

Other types of shear deformation.

Linear vs Nonlinear behavior



Linear regime for polymers



Oscillations at certain strain γ and frequency (and T).

What strain do we choose???

→ Dynamic Strain Sweep

Monodisperse polybutadiene melt
 $M_w = 99060$ g/mol at 40°C

Linear vs Nonlinear behavior

Linear measurements: Apply deformation (or stress) to probe structure, not to influence/change structure.

→ Structure independent of applied deformation

Nonlinear deformation (or stress) affects structure.

→ Type of deformation.

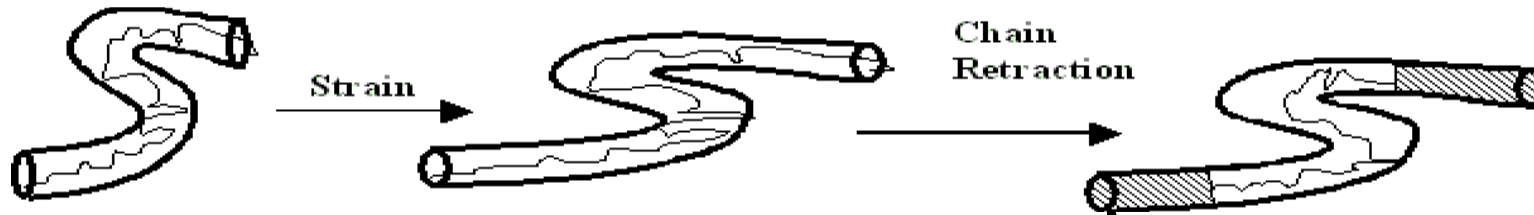
→ Deformation history.

Strain or stress control (here strain)

Polymers under nonlinear deformation

- Linear behavior = reptation, CLF, constraint release
- Orientation of the tube segments (rate > terminal relaxation)
- Stretch of the chain (rate > Rouse time)

$$Wi = \dot{\gamma}\tau$$

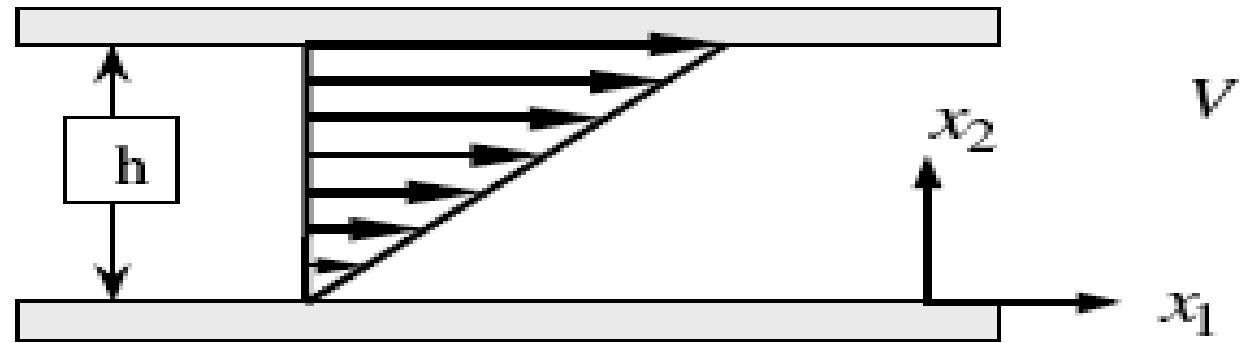


- Effects of flow-field: Convective constraint release (CCR)



Marrucci
Ianniruberto

How to obtain data on polymers in simple shear flow?

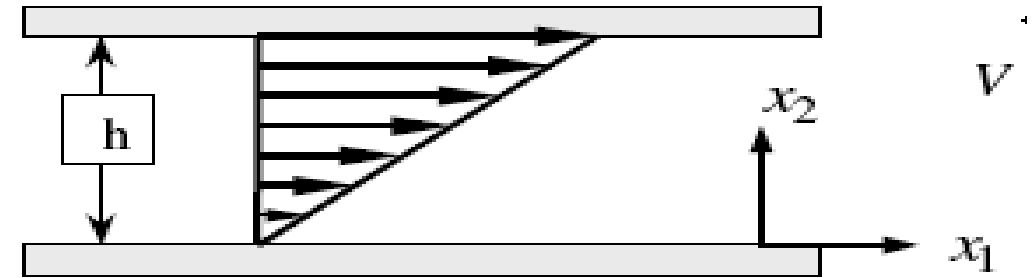


$$V_1(x_1) = f(x_2)$$

$$\frac{D\sigma_p}{Dt} = f(\sigma_p, \nabla \mathbf{v})$$
$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

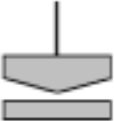
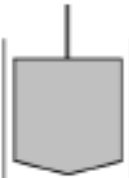
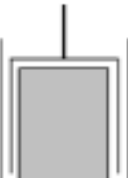
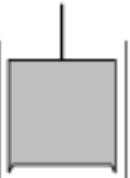
Simple shear flow

Sliding plate rheometer

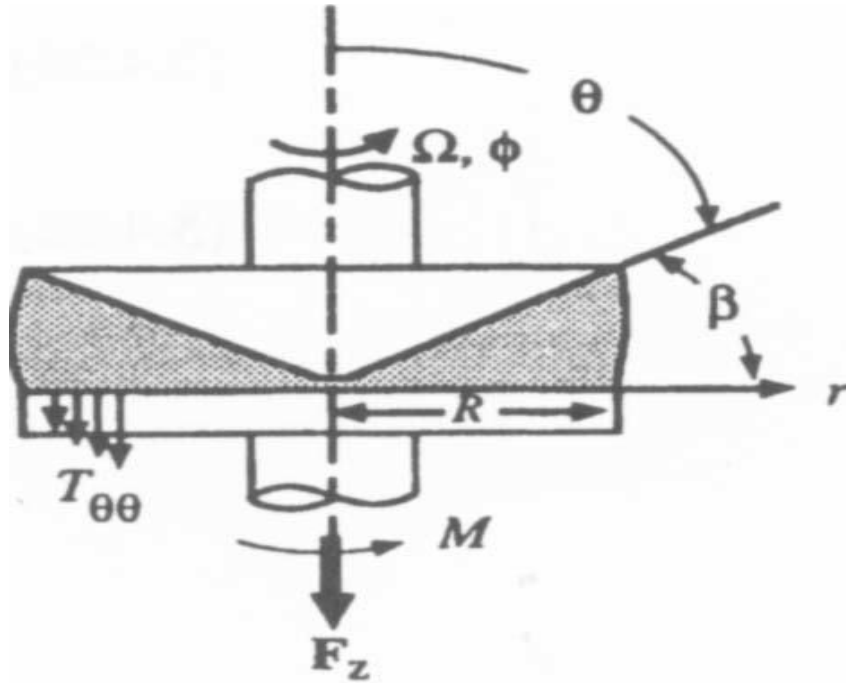


Ideal flow field but not commercially available (only home-made setups). Not on rotational rheometer.

Instabilities are a problem.

Cone-Plate	Plate-Plate	Coaxial Cylinder	Double Gap Cell	Mooney Cell	...
					...

Cone and plate



$$u_{\phi}(r, \theta) = rf(\theta)$$

$$\gamma = \frac{\Omega}{\beta}$$

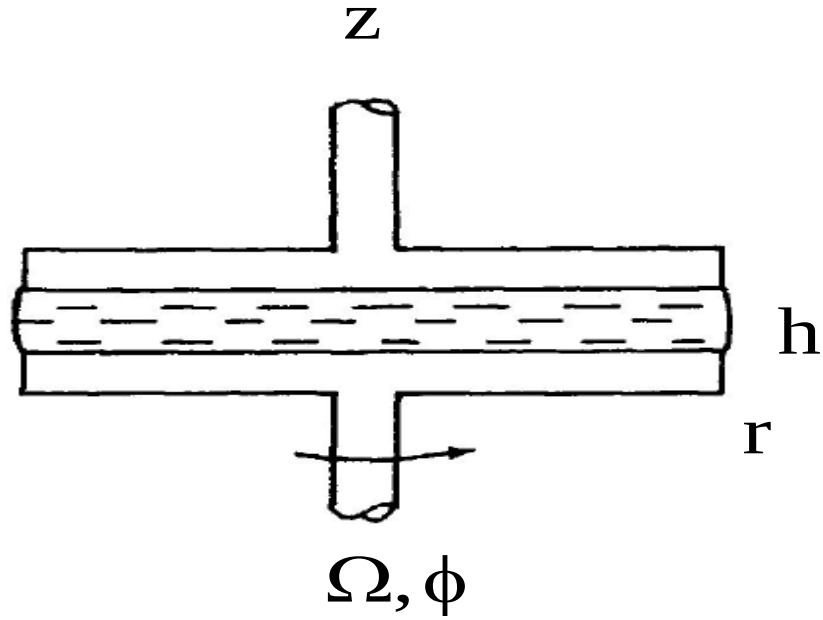
$$\dot{\gamma} = \frac{\dot{\phi}}{\beta}$$

$$\sigma_{\phi\theta} = \frac{3M}{2\pi R^3}$$

$$\beta < 0.1 \text{ rad}$$

- Readily commercially available for rotational rheometers
- In principle ideal simple shear flow
- Prone to instabilities (i.e. slip, edge fracture)
- Ok for not too elastic samples at low rates

Parallel plates



$$u_{\phi}(r, z) = rf(z)$$

$$\gamma = \frac{\phi r}{h}$$

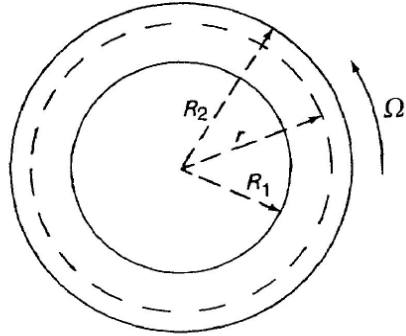
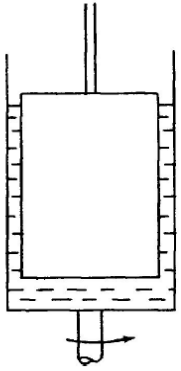
$$\dot{\gamma} = \frac{r\Omega}{h}$$

$$\sigma_{\phi z} = \frac{M}{2\pi R^3} \left(3 + \frac{d \ln M}{d \ln \dot{\gamma}_R} \right)$$

Good for LVE

Careful with NLVE: shear rate is not constant

Concentric cylinders (Couette)



$$\kappa = \frac{R_1}{R_2} \geq 0.99$$

Best for low-viscosity fluids

$$\frac{v_\theta}{r} = \Omega \frac{\left(\frac{1}{r^2} - \frac{1}{R_1^2}\right)}{\left(\frac{1}{R_2^2} - \frac{1}{R_1^2}\right)}$$

$$\dot{\gamma} = \left| r \frac{\partial(v_\theta/r)}{\partial r} \right| = \frac{2\Omega}{r^2 \left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)}$$

$$R_1 \cong R_2 \cong \bar{R}$$

$$\dot{\gamma} \cong \frac{\Omega \bar{R}}{(R_2 - R_1)}$$

$$\gamma = \frac{\theta \bar{R}}{R_2 - R_1}$$

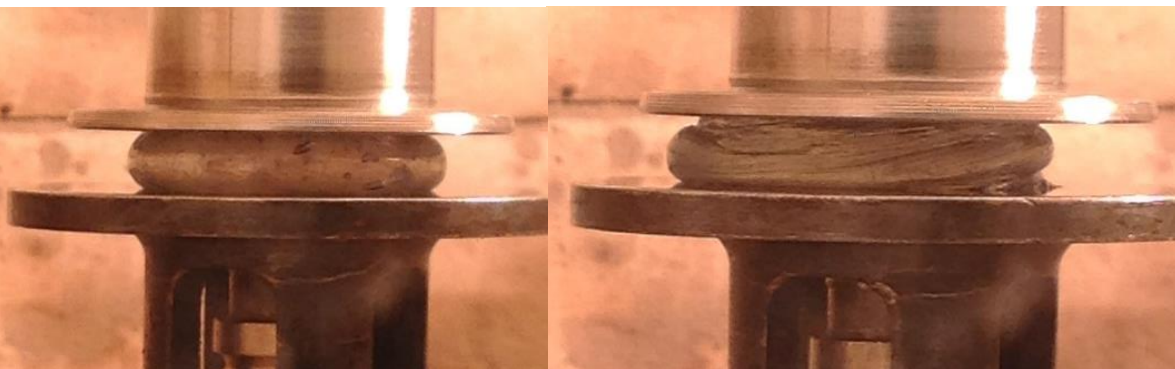
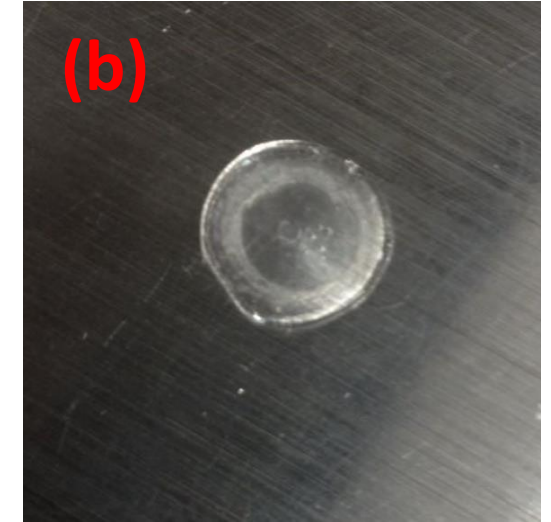
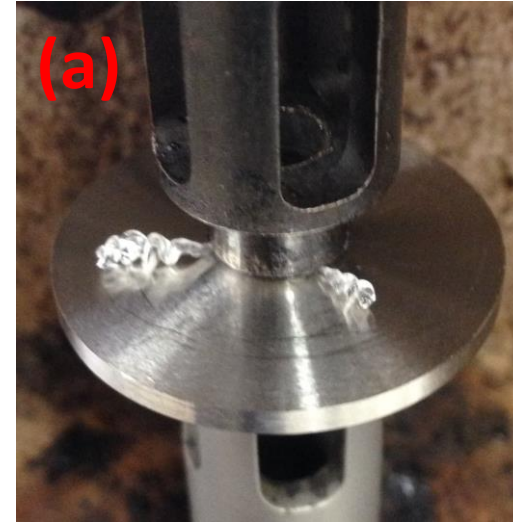
Shear measurements: fracture & slip



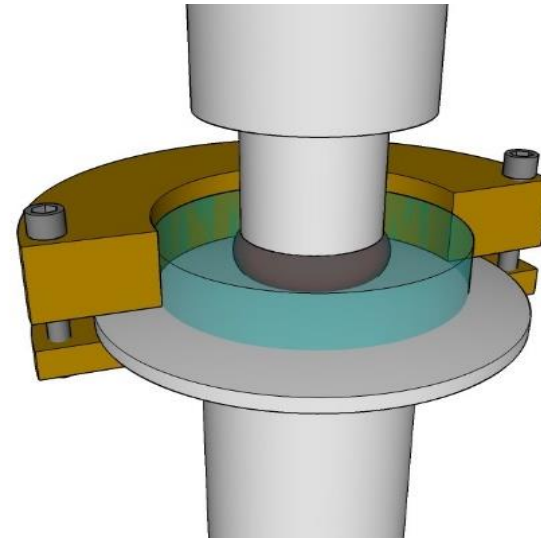
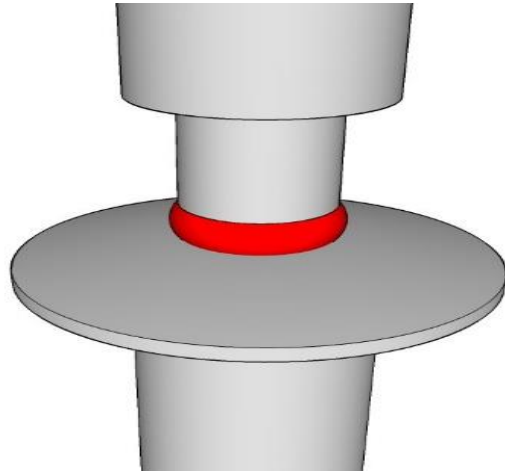
Shear measurements: fracture & slip



Shear measurements: fracture & slip



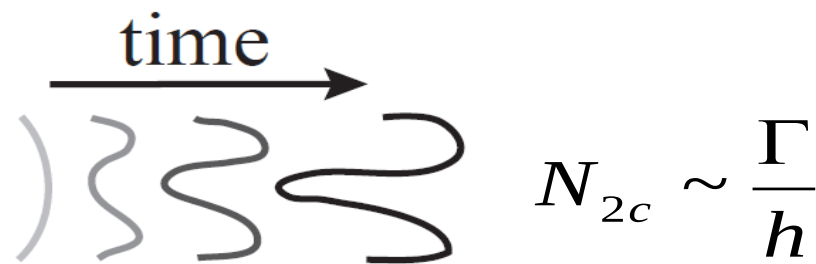
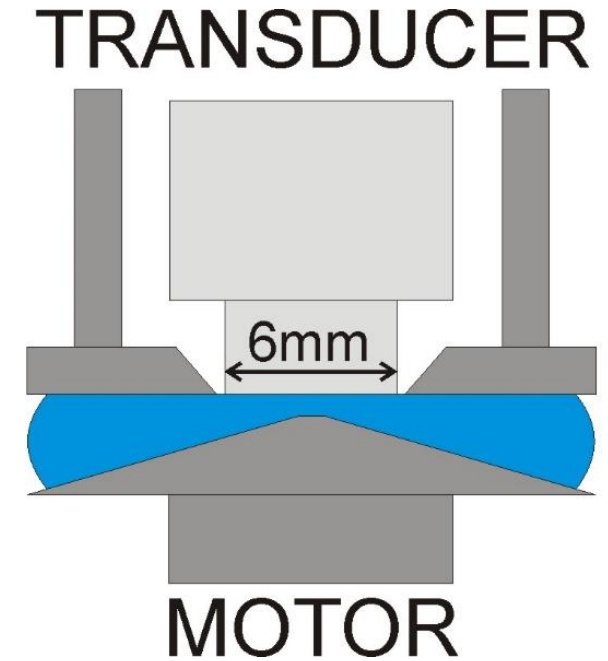
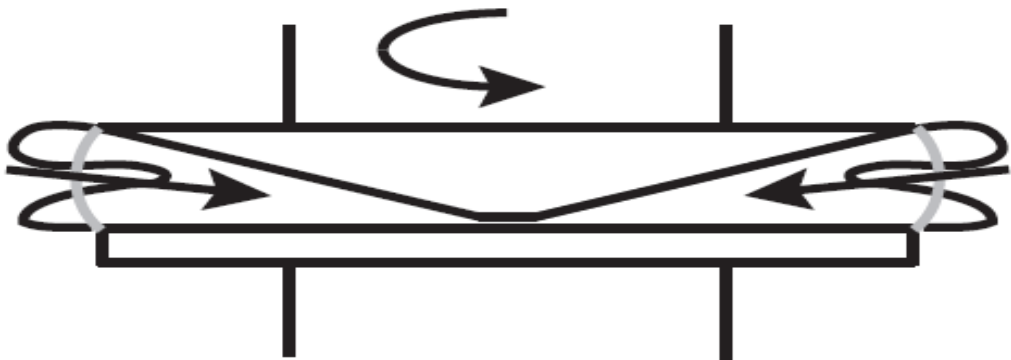
Reduce / eliminate fracture ?



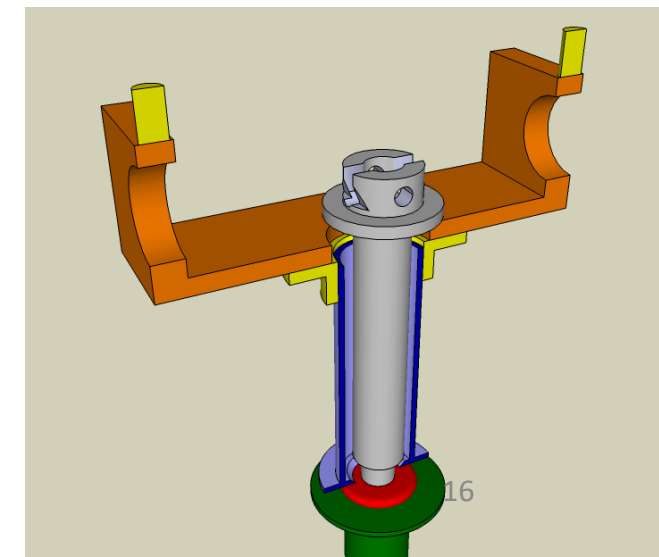
Measurements with Cone-partitioned plate (CPP) geometry

Pollet + Cross, J. Sci. Instrum. 1950
 Meissner et al., J. Rheol. (1989)
 Ravindranath + Wang, J. Rheol. (2008)
 Schweizer et al., JOR 2013
 Skorski and Olmsted, J. Rheol. 2011

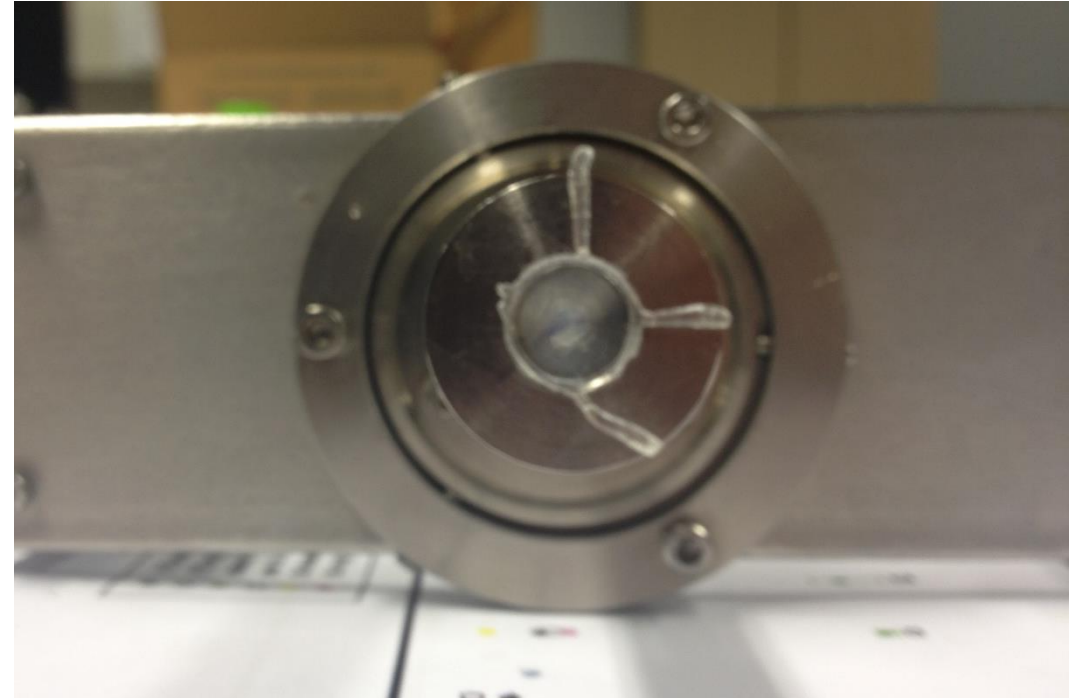
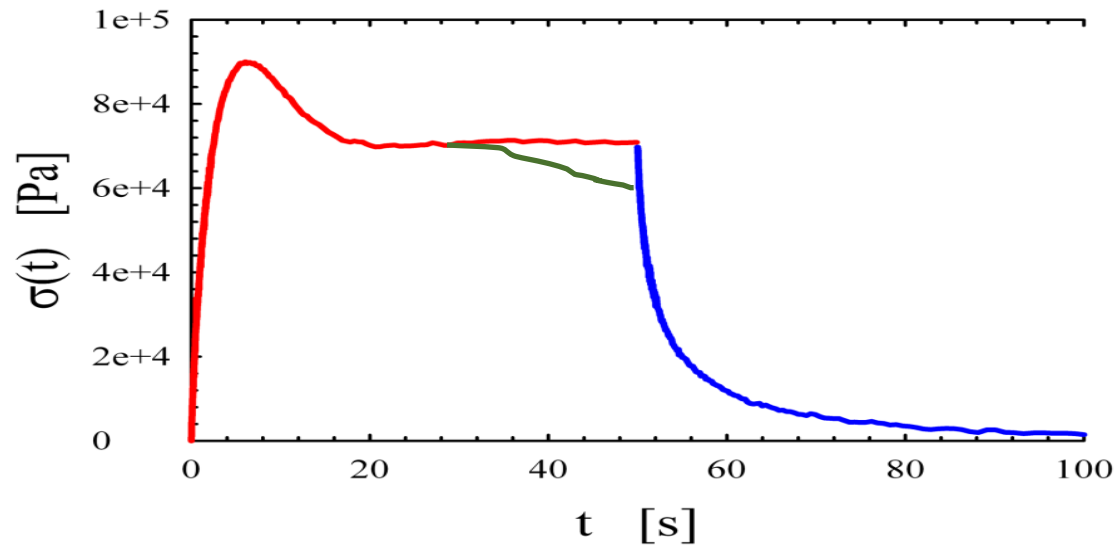
Tanner + Kenntok, JOR 1983
 Schweizer et al., J. Rheol. (2003-6)
 Snijkers, DV, JOR 2011
 Costanzo et al. Macromolecules 2016



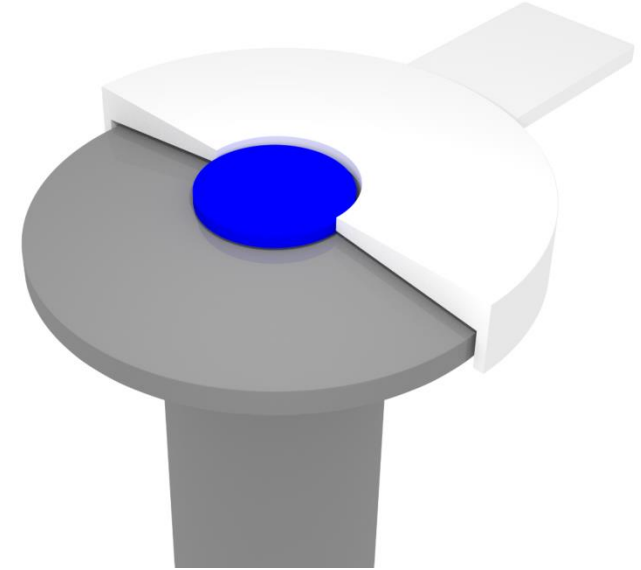
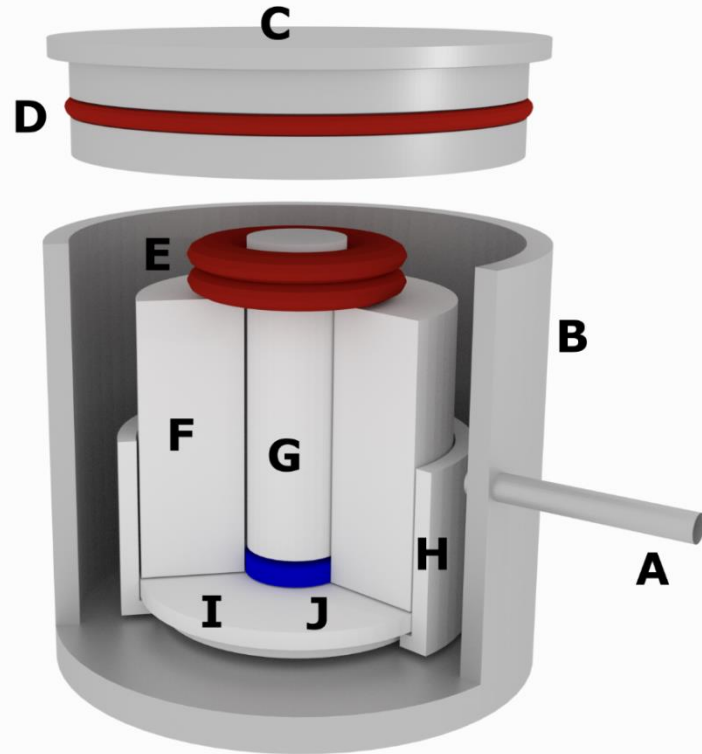
Fracture does not
 influence
 measurement



CPP and edge fracture

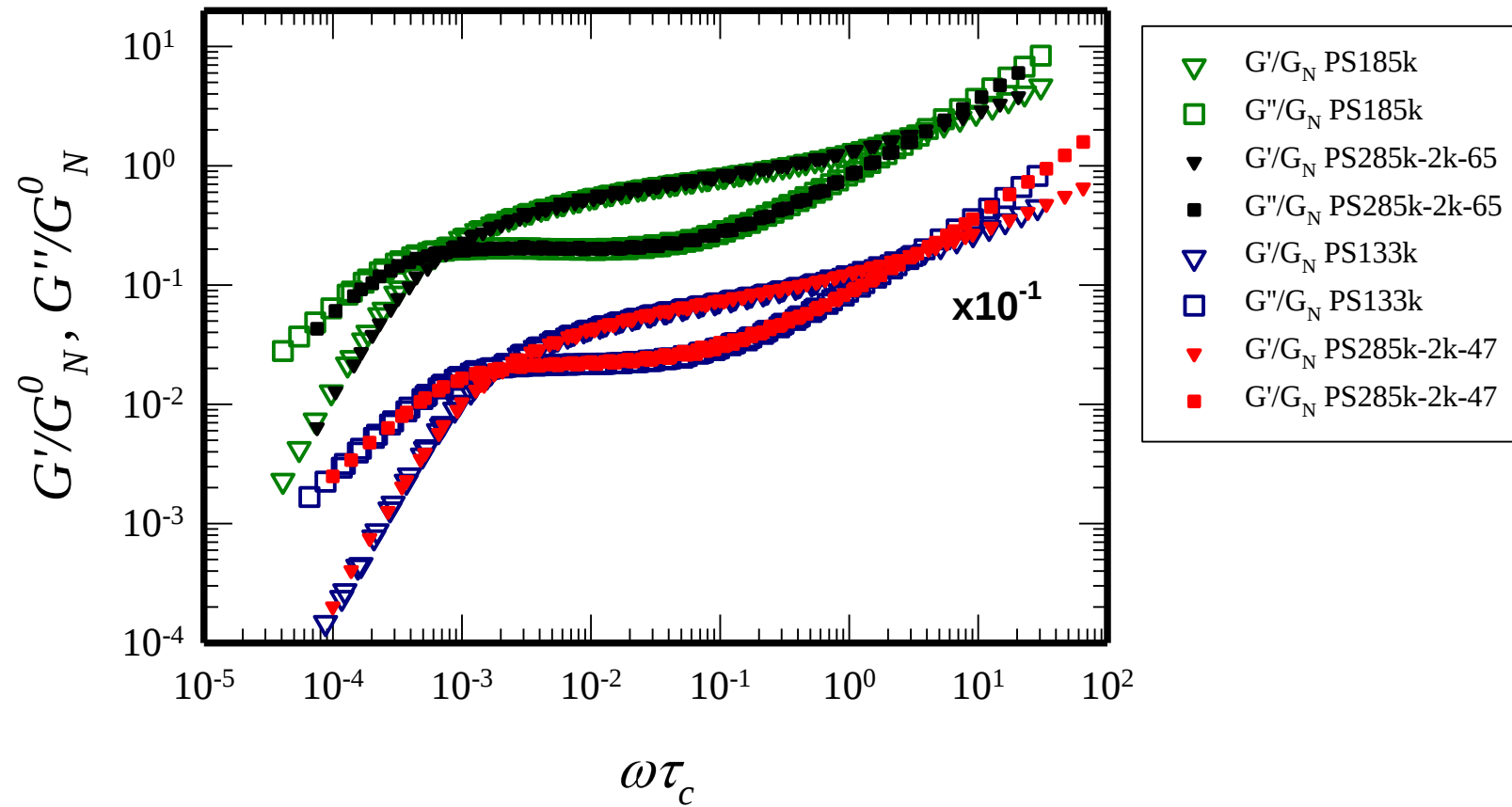


Sample handling



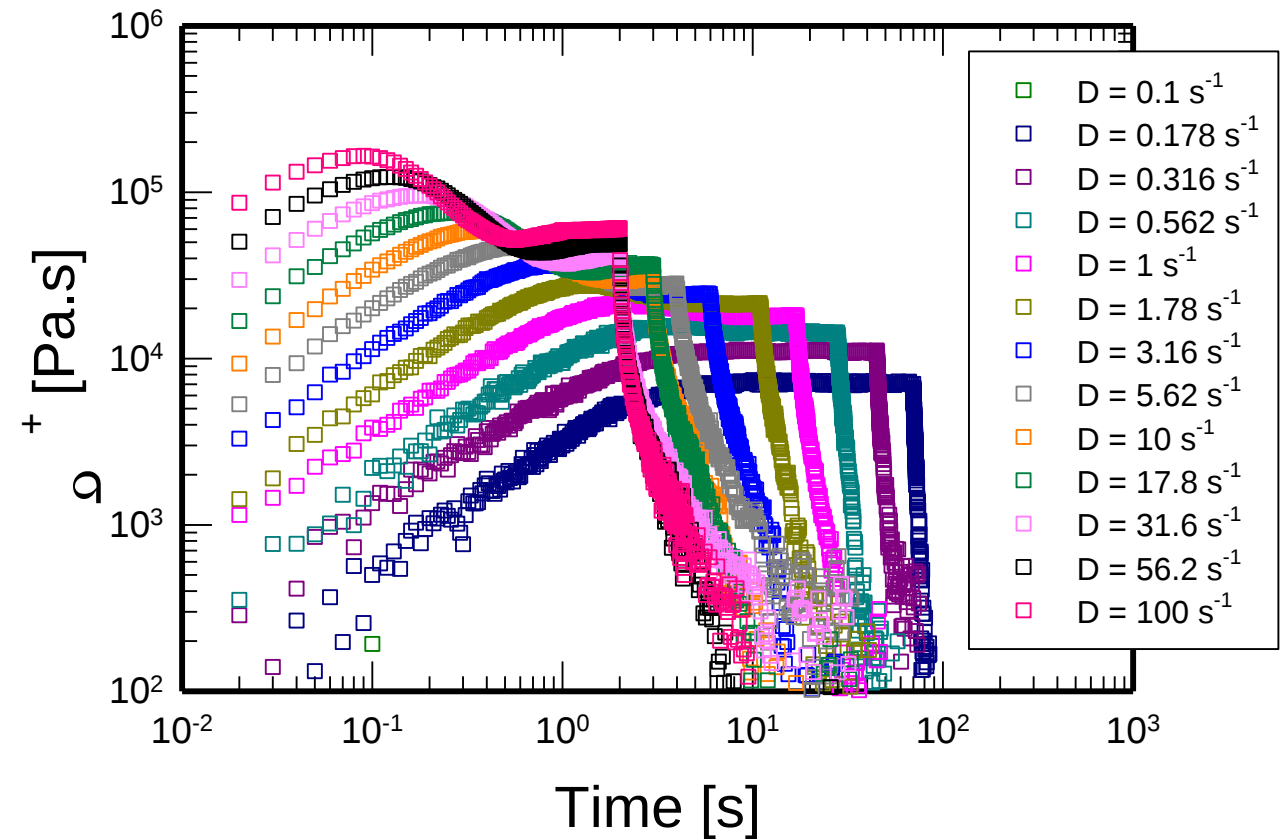
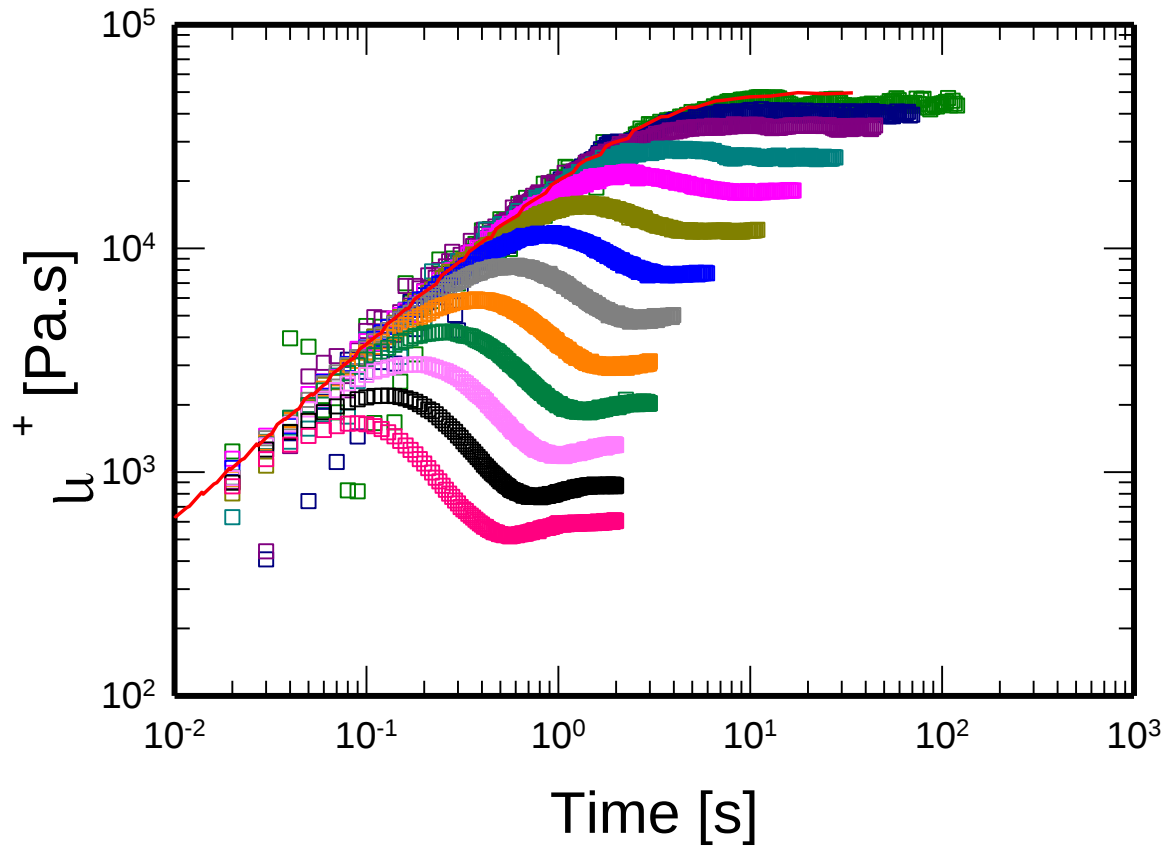
T_g is a determining factor

LVE scaling

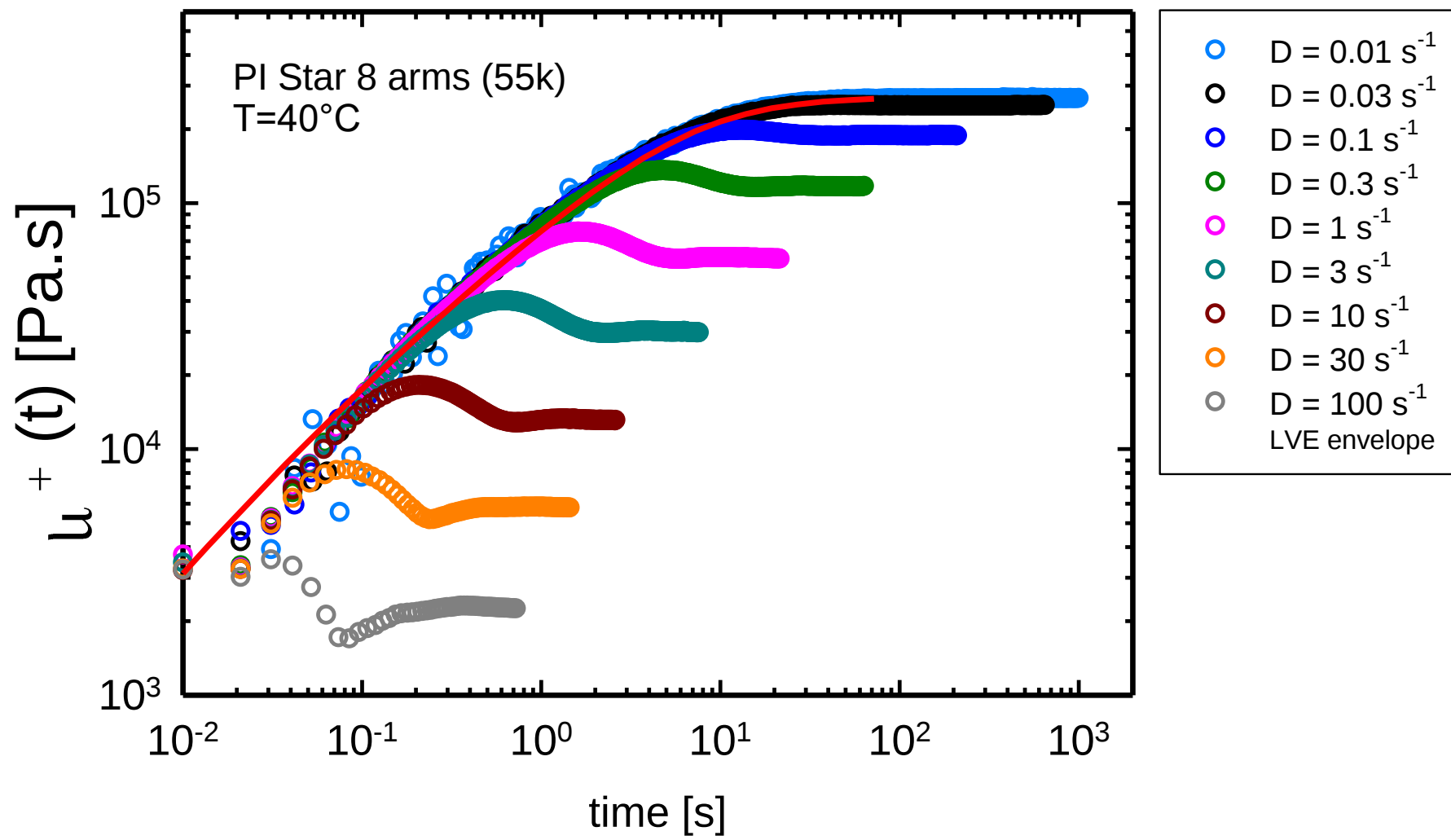


Start-up and relaxation in simple shear

PS285k-2k-47 @ 140°C

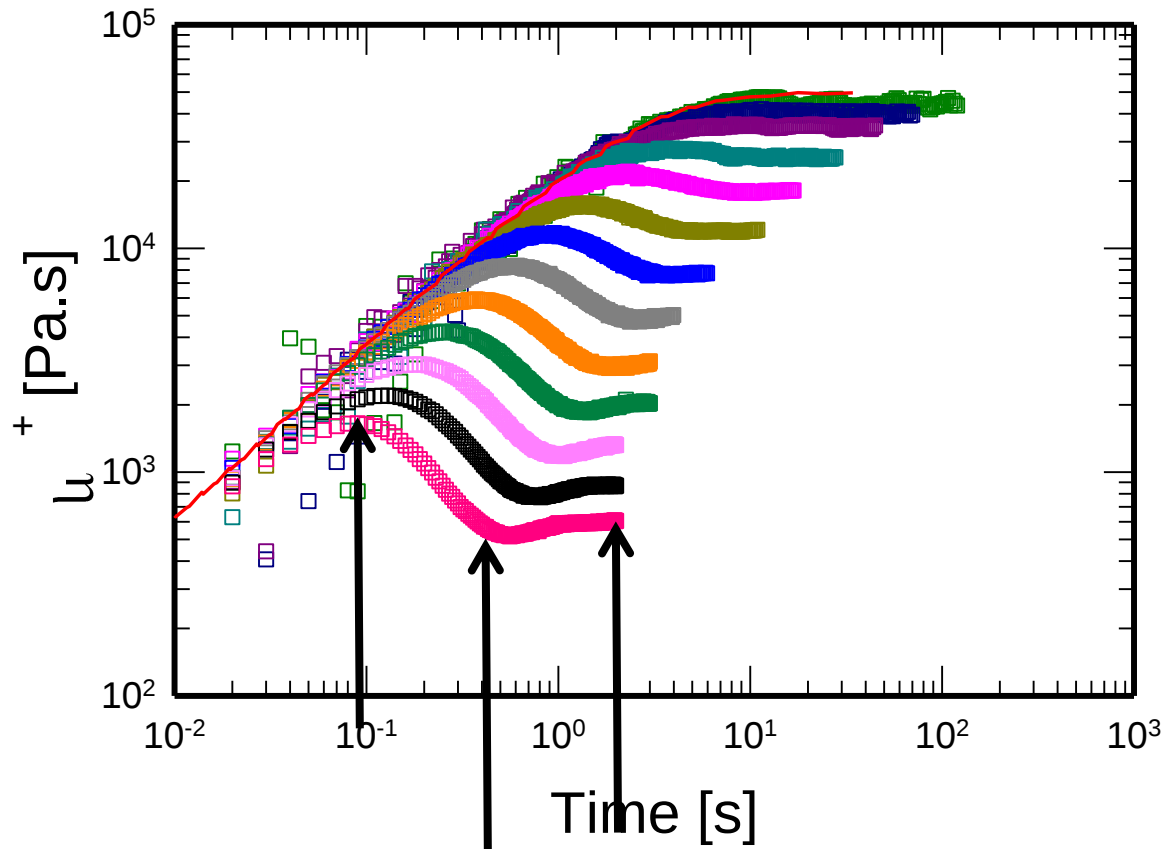


No steady state shear banding
Undershoot at large shear rates



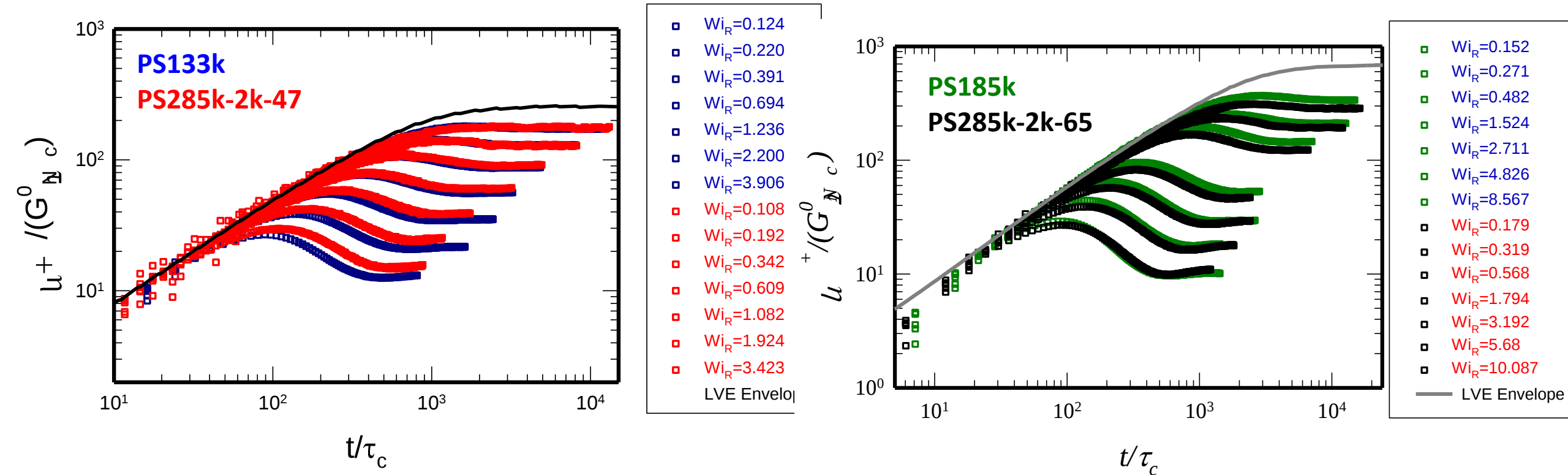
Start-up and relaxation in simple shear

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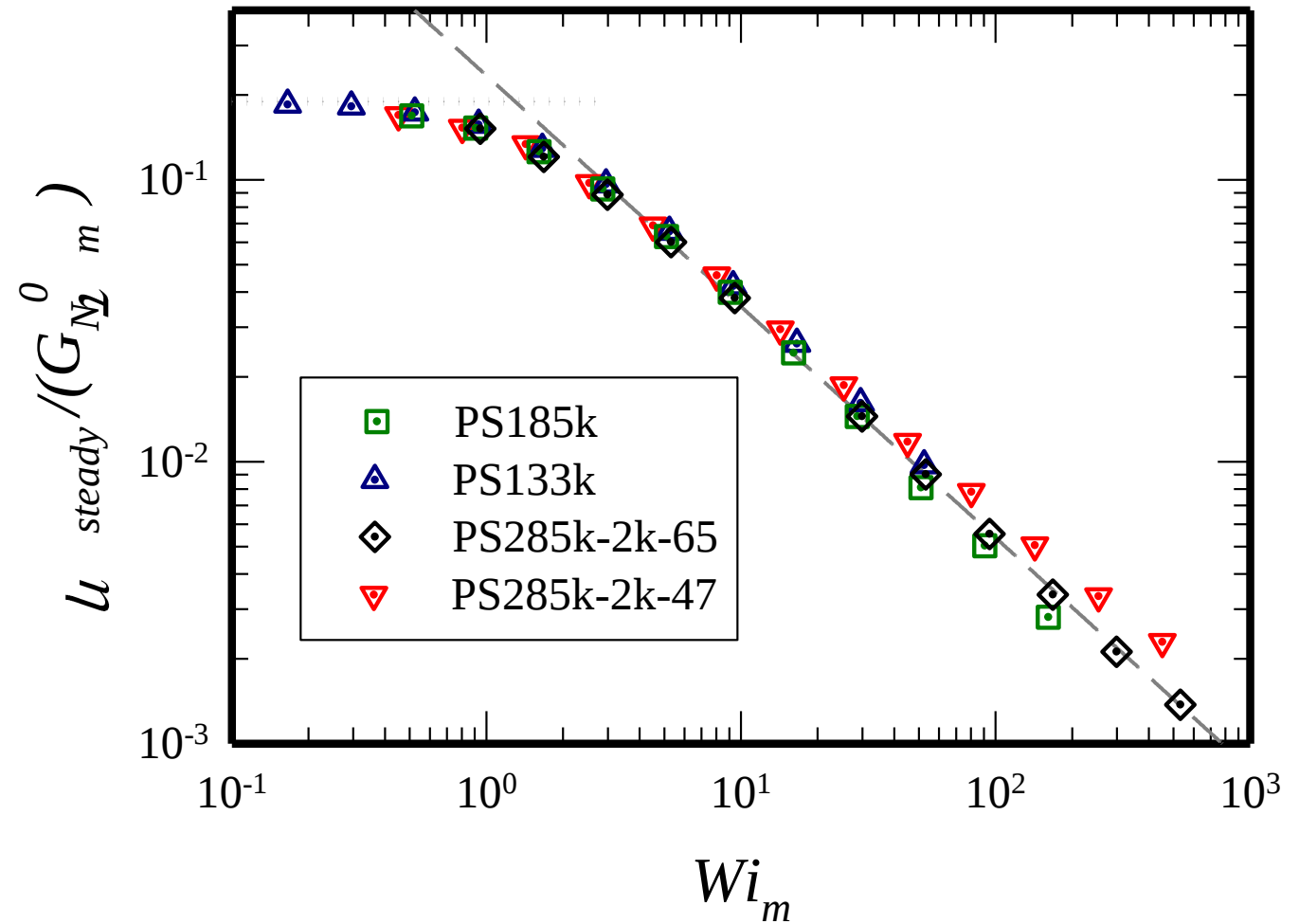
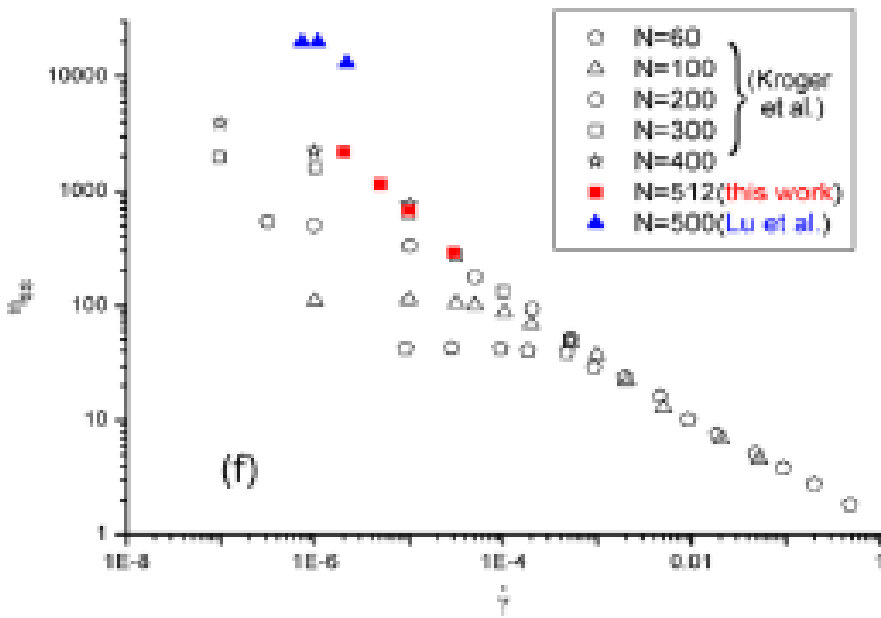
Nonlinear Start-up shear flow: scaling



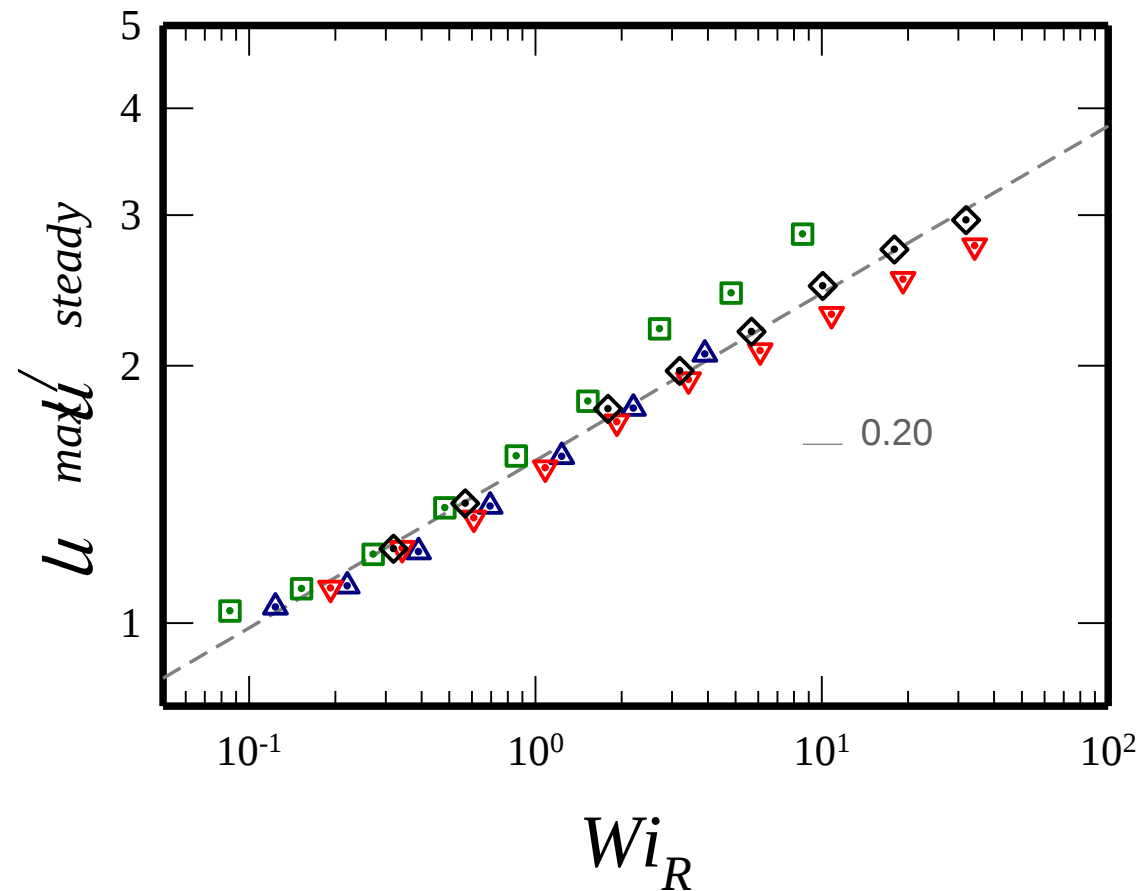
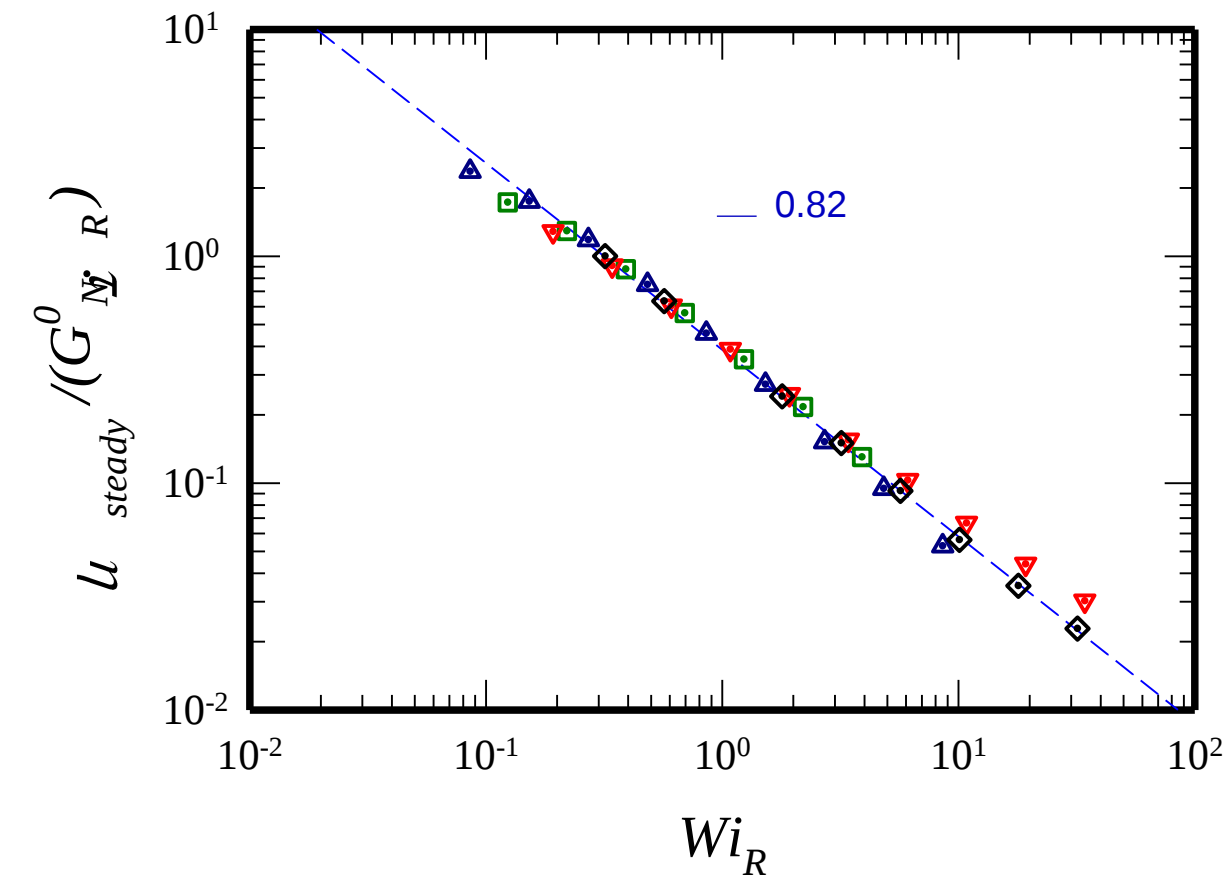
No difference between PS melts and solutions with the same Z

Steady shear flow

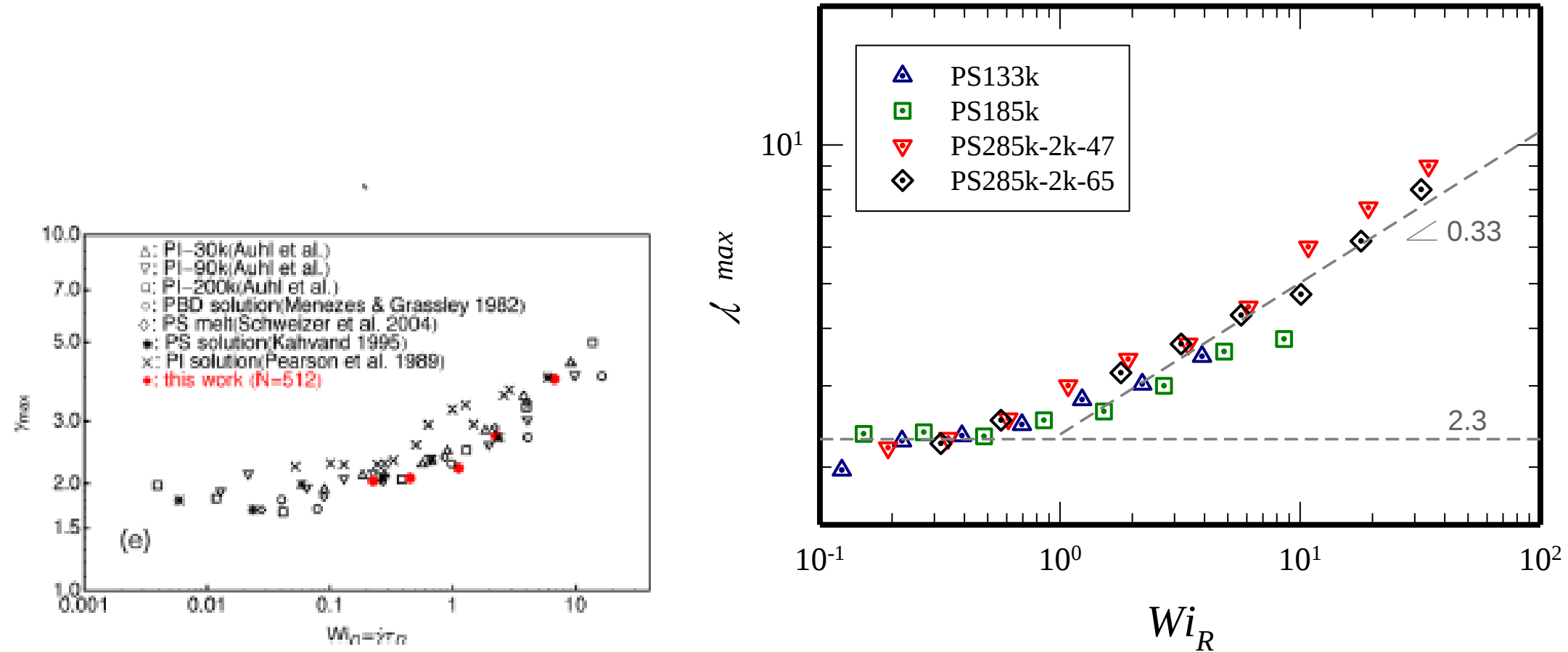
Thinning slope: -0.82



Shear viscosity: scaling

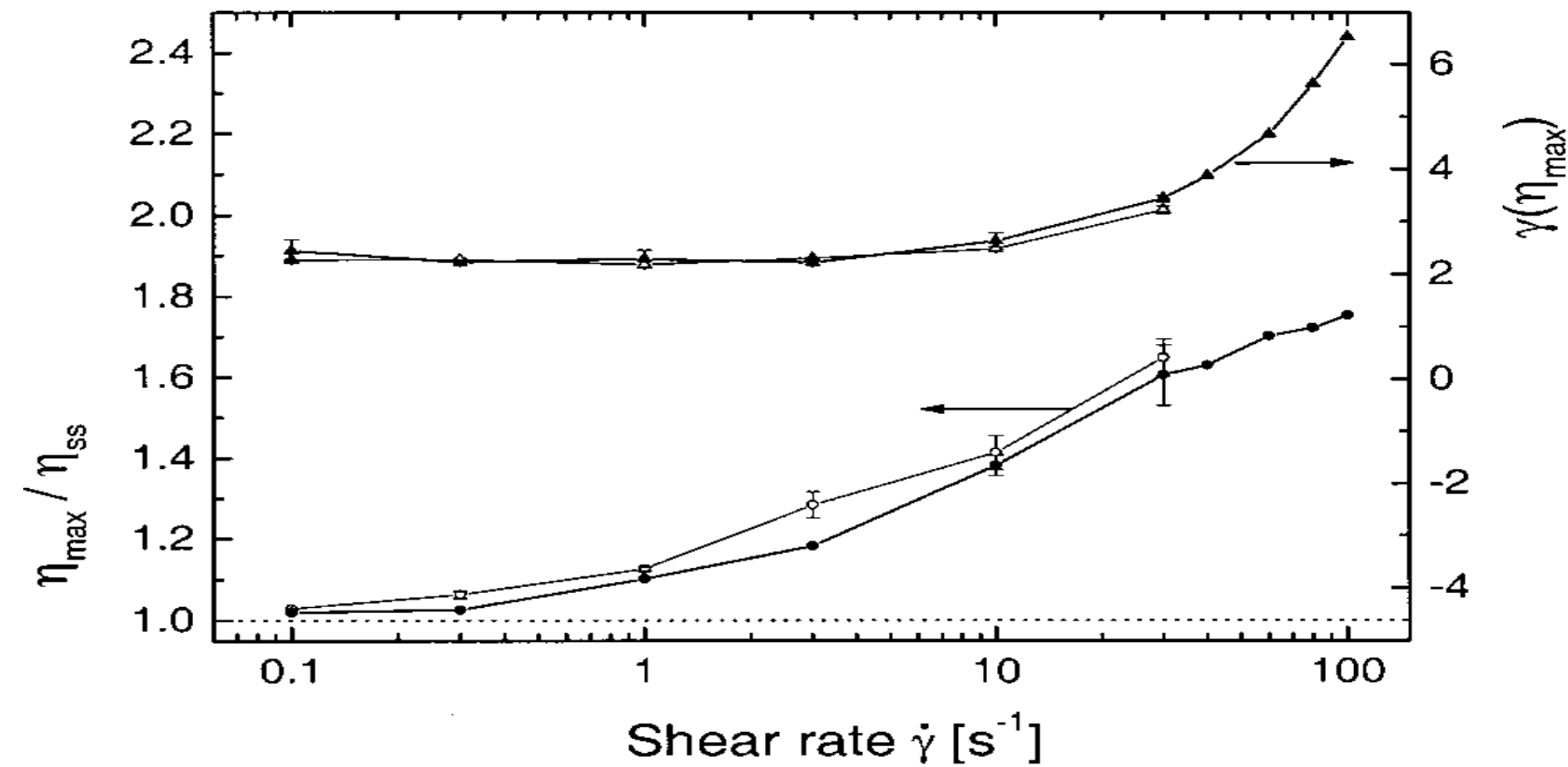


Peak strain: scaling



GLaMM: Graham et al., J. Rheol. 2003 (1)
 K. S. Schweizer (2015) : retraction + internal force (1/3)
 Cao and Likhtman , ACS Macro Lett. 2015 (1/3)
 Auhl et al., J. Rheol. 2008

Ravindranath and Wang, J. Rheol. 2008
 Schweizer et al., J. Rheol. 2004
 Snijkers and DV, J. Rheol. 2011
 Costanzo et al., Macromolecules 2016



[Linear melt but
not monodisperse]

Comparing cone-partitioned plate and cone-standard plate shear rheometry of a polystyrene melt

Th. Schweizer^{a)}

Institute of Polymers, ETH Zürich, CH-8092 Zürich, Switzerland

(Received 20 February 2003; final version received 14 April 2003) 27

Molecular modeling (Ianniruberto+Marrucci)

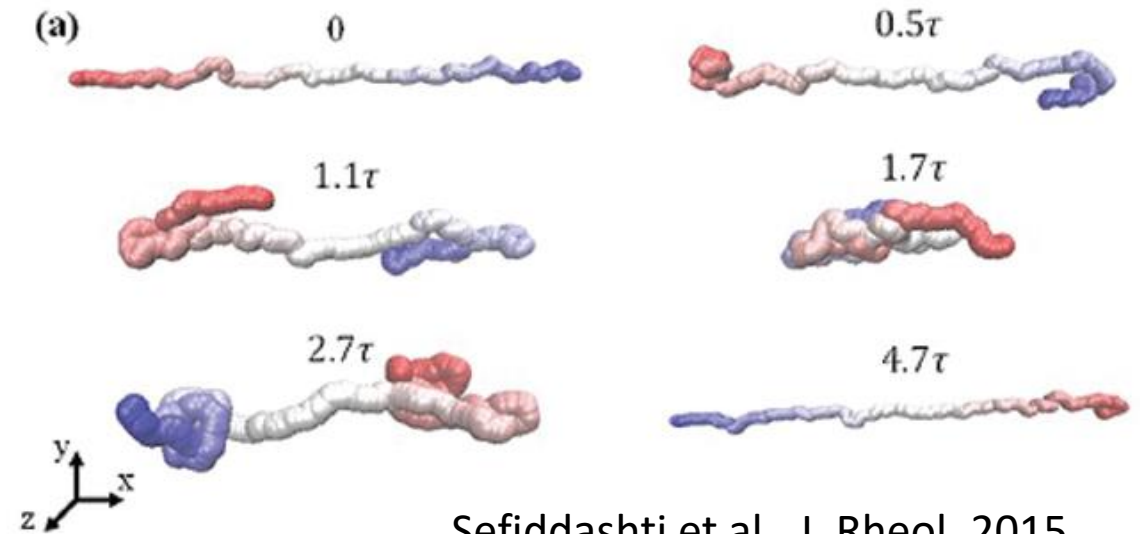
Decoupling stretch and orientation

$$\sigma(t) = C_Q \lambda^2(t) \frac{f(\lambda)}{f_0} \sum G_i S_i$$

Stretch ratio

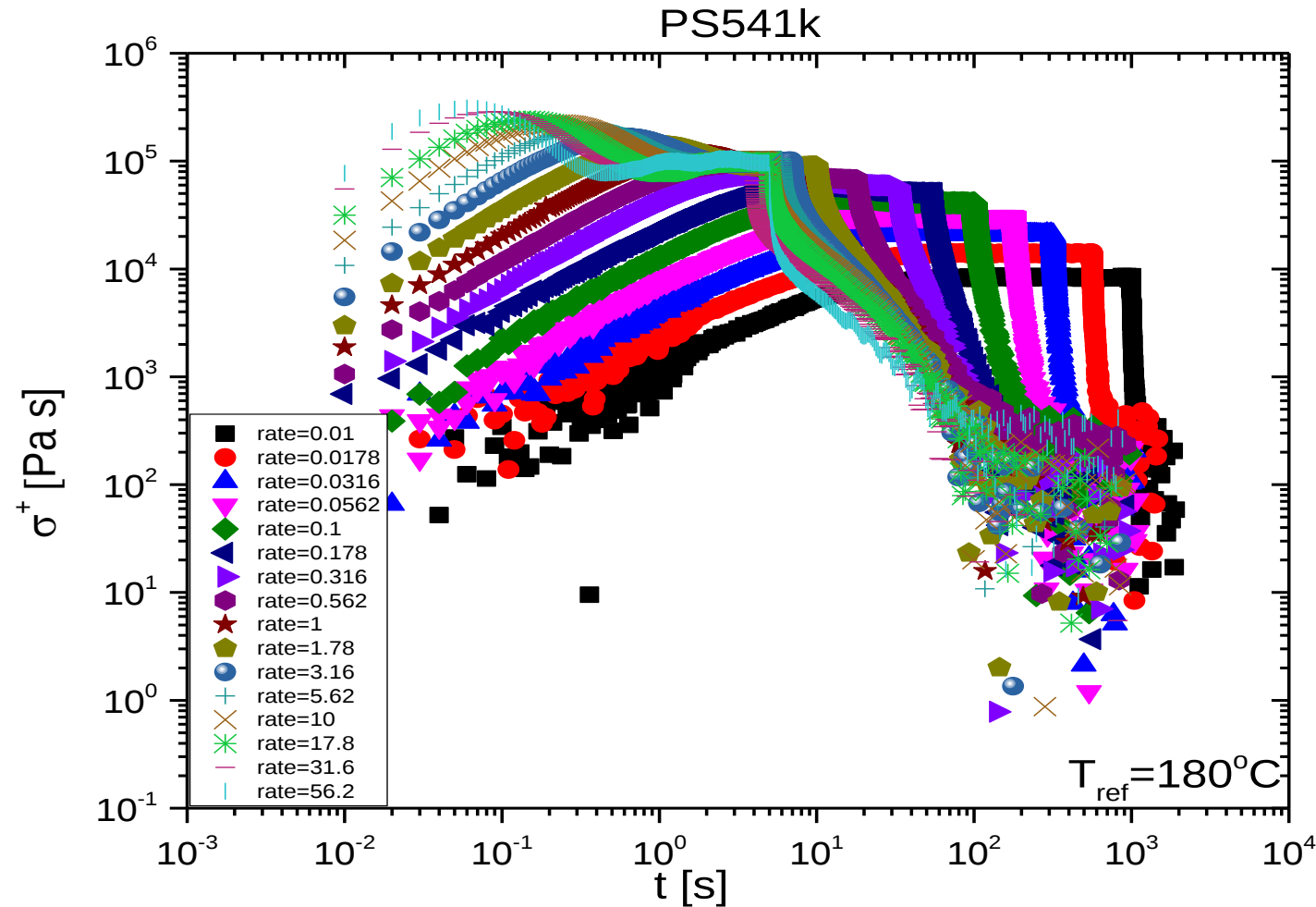
Friction reduction in elongation

Tumbling function in shear

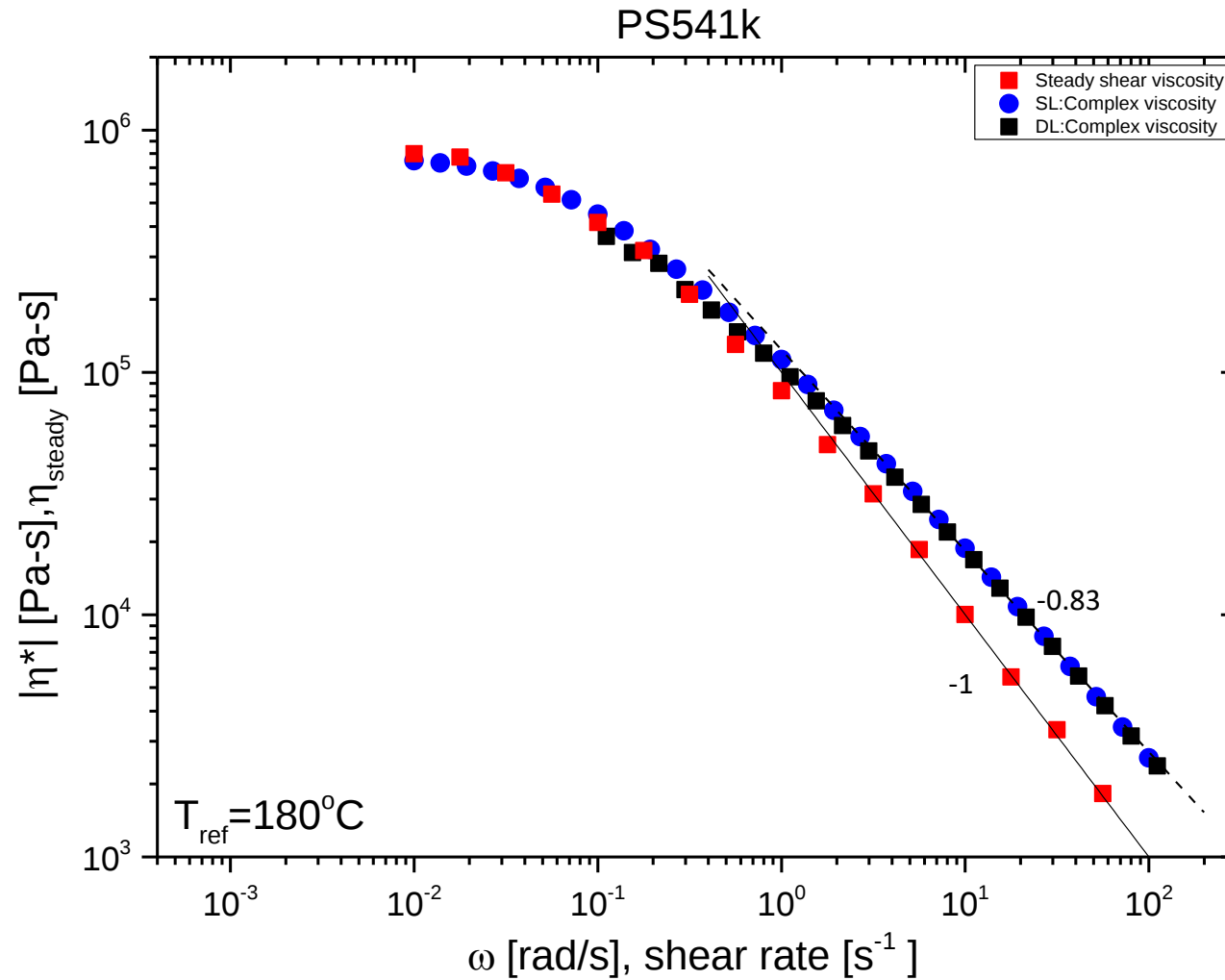


Sefiddashti et al., J. Rheol. 2015

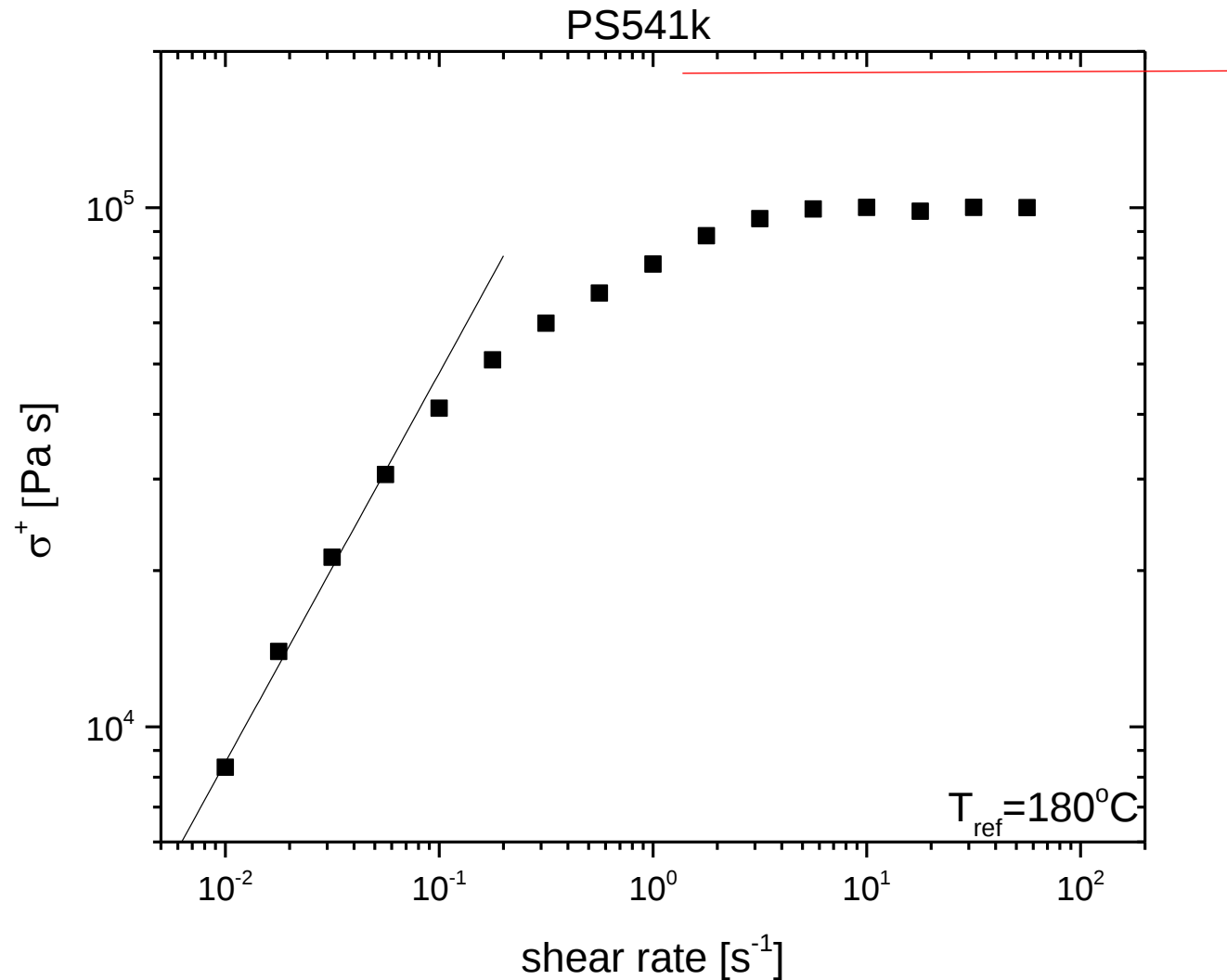
Careful: possible issues with data



Careful: possible issues with data



Careful: possible issues with data



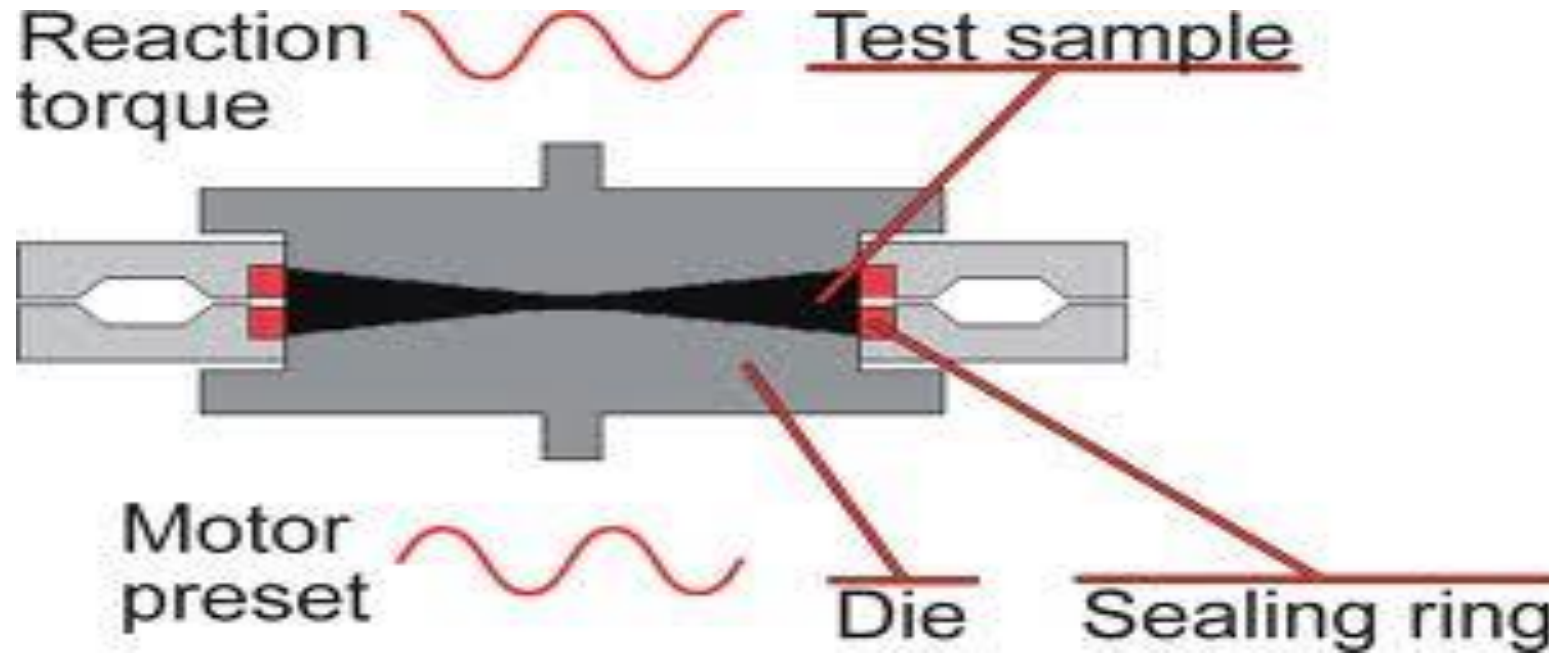
Shear banding

P. D. Olmstead
S. Fielding
W. J. Briels

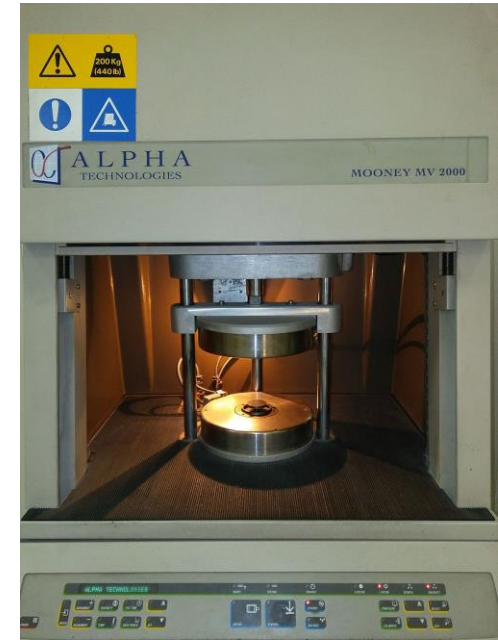
Higher rates

- Why do we need high rates? [$Wi_t > 10^3$]
- Above setups: Roughly ideal flow fields, but limited in rate, mainly due to instabilities especially for highly elastic samples but also due to technical limitations.
- How to reach higher rates?
 - Mooney viscometer
 - Capillary rheometer
 - Less ideal shear field... Hence more data analysis needed which includes sometimes dubious assumptions.

Mooney viscometer



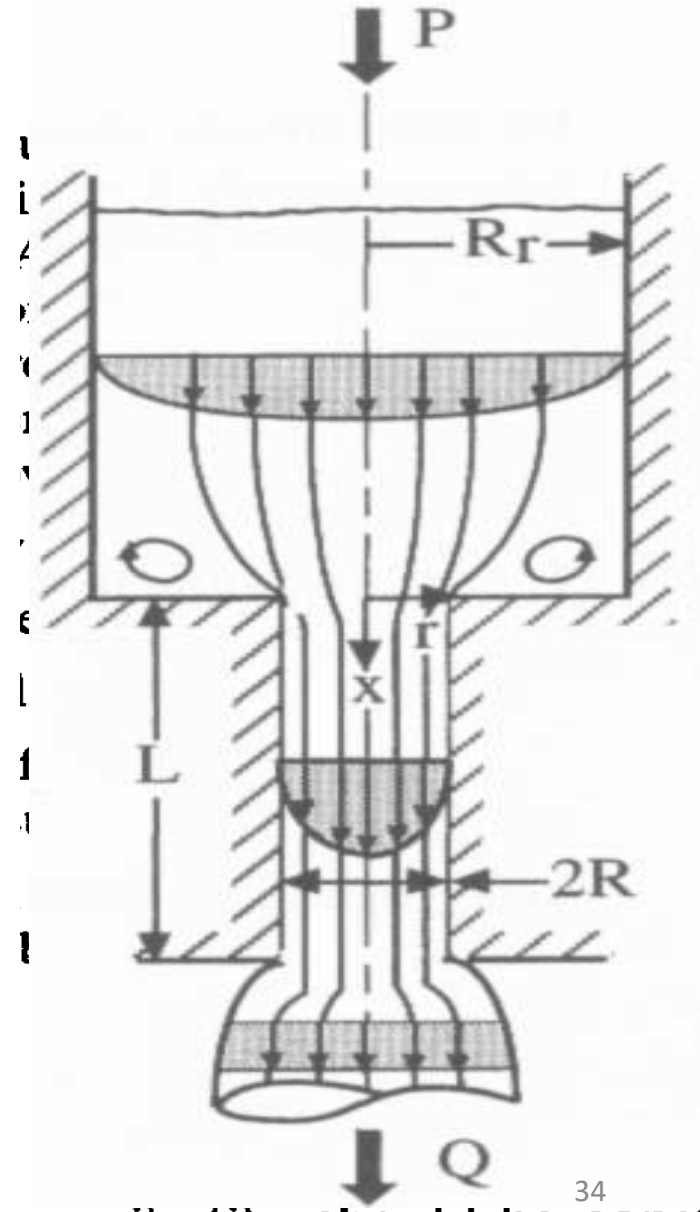
- Biconical shape
- Closed chamber
- Under pressure (→ less instabilities)
- Typical for rubber testing (mainly industry)



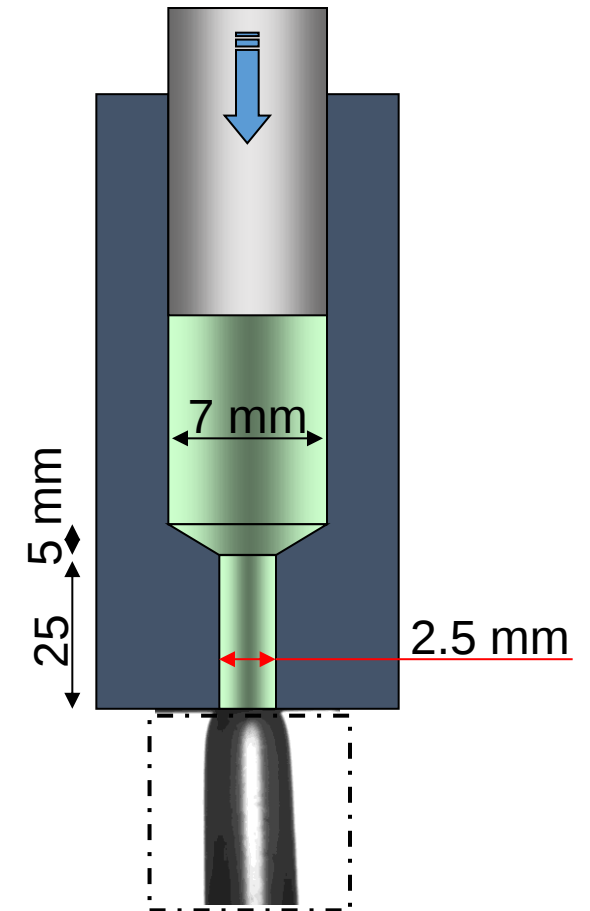
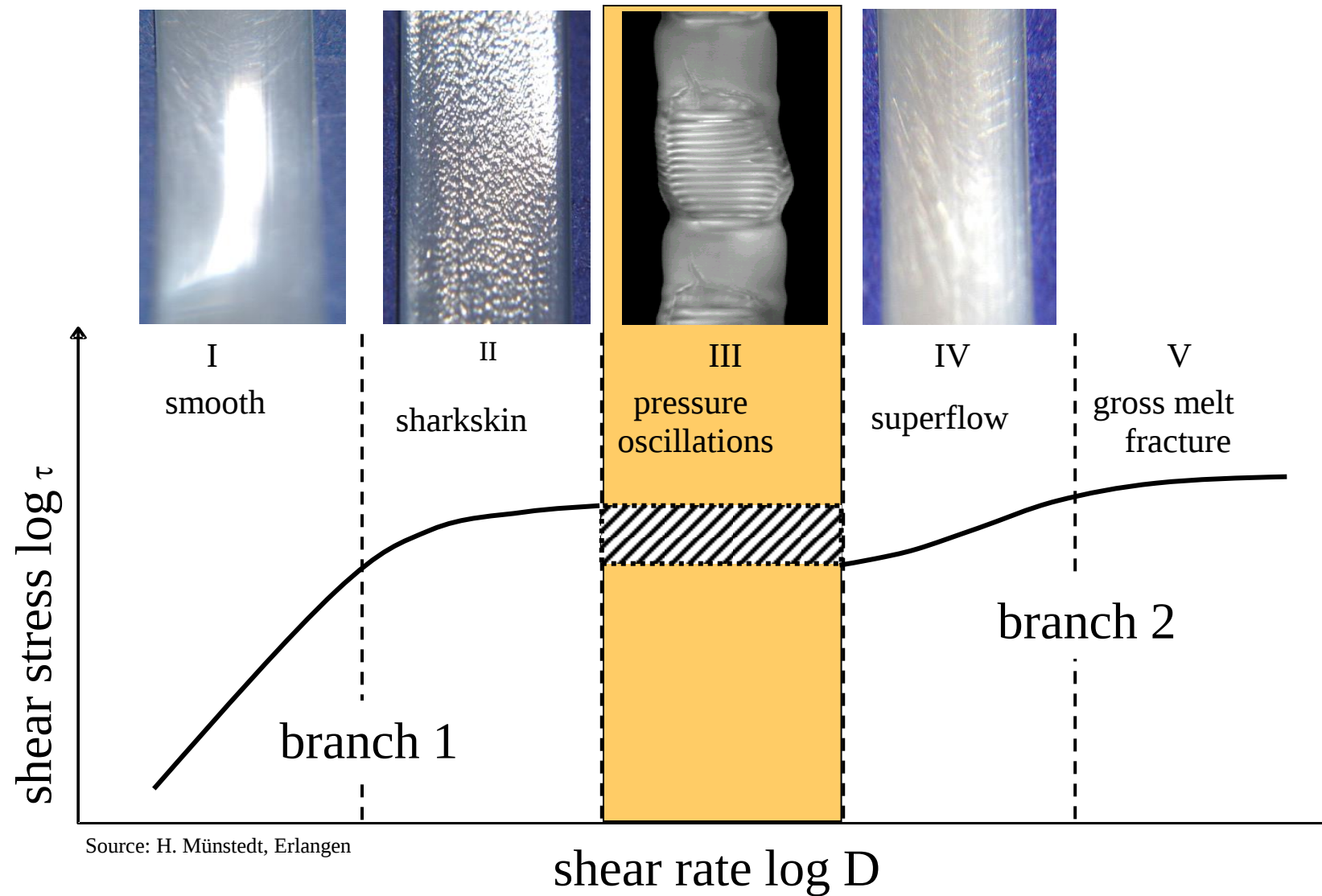
Macosko 1994

Capillary rheometry

- Poiseuille flow (not simple shear) – parabolic profile
- Entrance and exit effects (Bagley)
- Very high rates
- Pressure effects
- Only steady properties
- Also possible instabilities

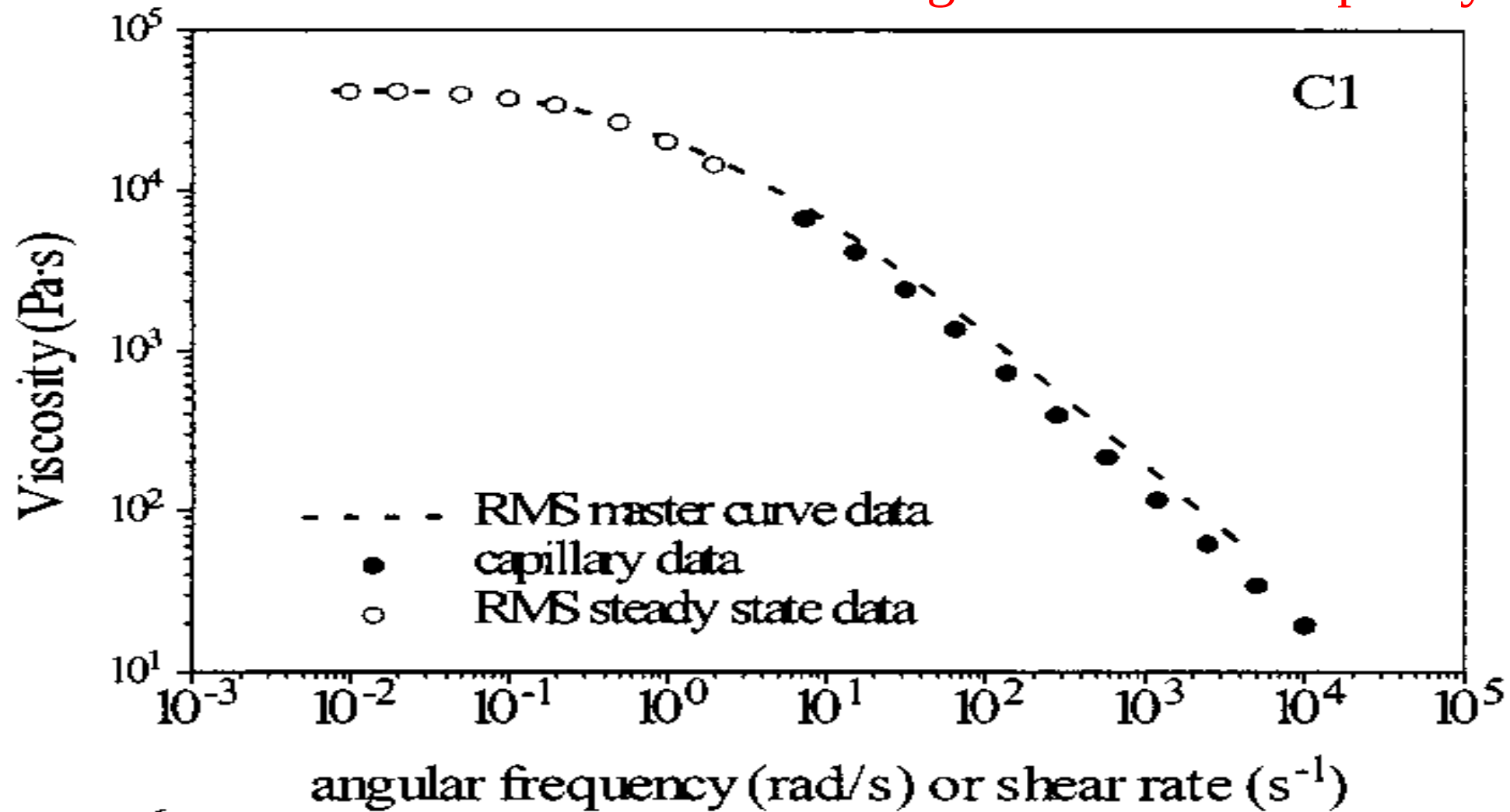


Examples of melt fracture for an HDPE



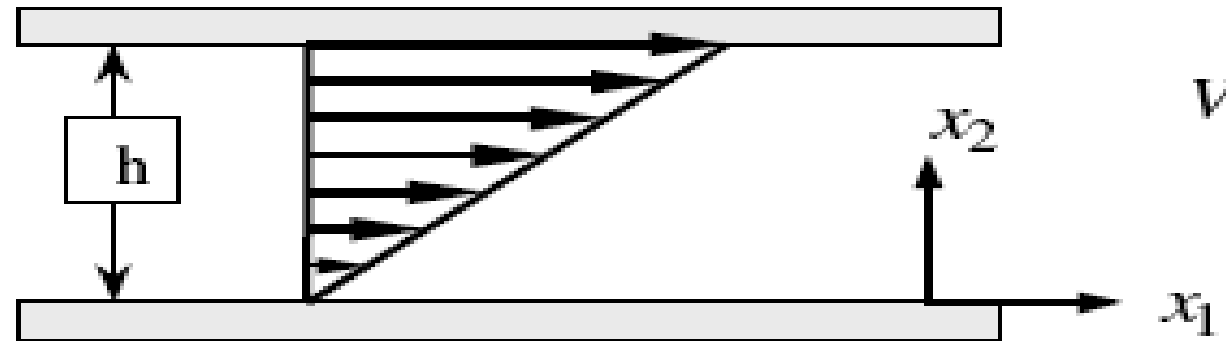
Example: Linear PS, $M_w = 300K$, PDI = 1.8

Higher rates with capillary rheometer



Behavior of polymers in simple steady shear flow

Basic observations



Observation 1: Polymers are shear-thinning

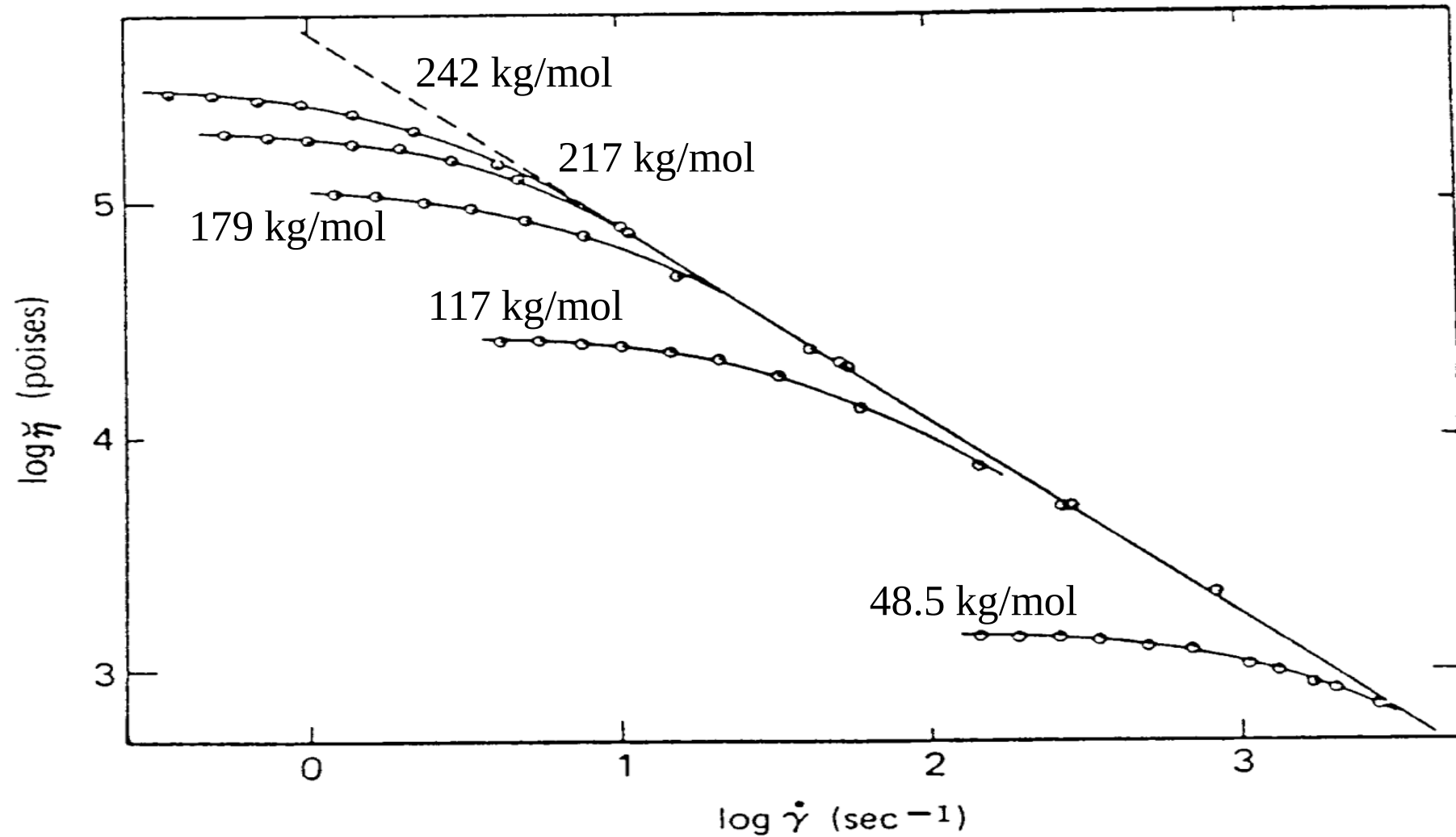


FIG. 13-9. Non-Newtonian viscosity η plotted against shear rate, for narrow-distribution polystyrenes.⁴⁵ Molecular weights from top to bottom, $\times 10^{-4}$: 24.2, 21.7, 17.9, 11.7, 4.85.

$$\log\left(\frac{h}{h_0}\right)$$

INDEPENDENT OF MW

(for entangled polymers)

Transition region

1

IDEALIZED VIEW

-1

(...roughly... -0.9x)

???

1

$$\log(Wi_0) = \log\left(\frac{\dot{\gamma}.t_0}{\lambda}\right)$$



Convective constraint release

Marrucci, JNNFM (1996) ; ; Ianniruberto JoR 2016

Ianniruberto + Marrucci JoR 2015

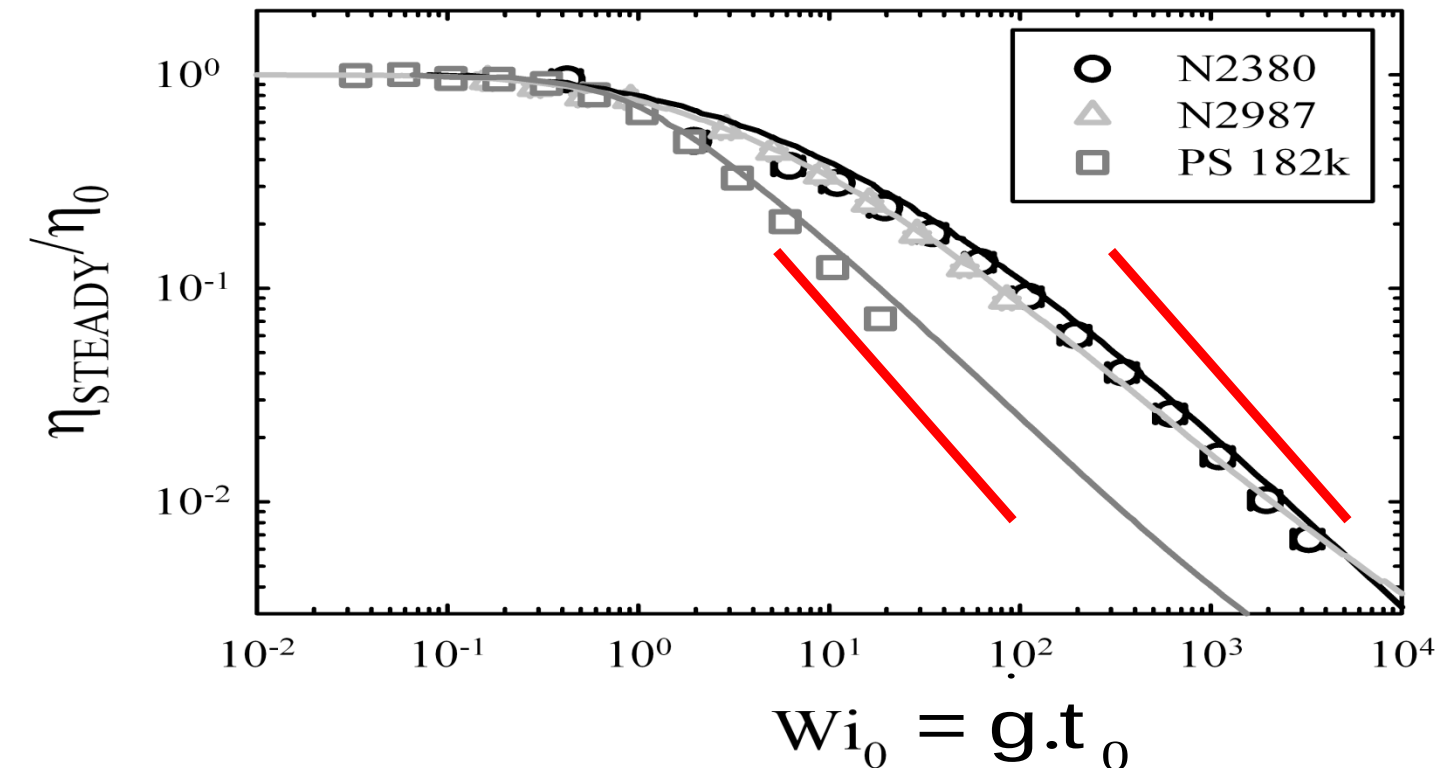
Observation 1: Polymers are shear-thinning

Observation 2: Linear polymers obey the empirical Cox-Merz rule

Cox-Merz (1958) rule:

$$\eta(\dot{\gamma}) = \eta^*(\omega) \Big|_{\omega=\dot{\gamma}}$$

Observation 3: Higher levels of polydispersity $\rightarrow$ less thinning.



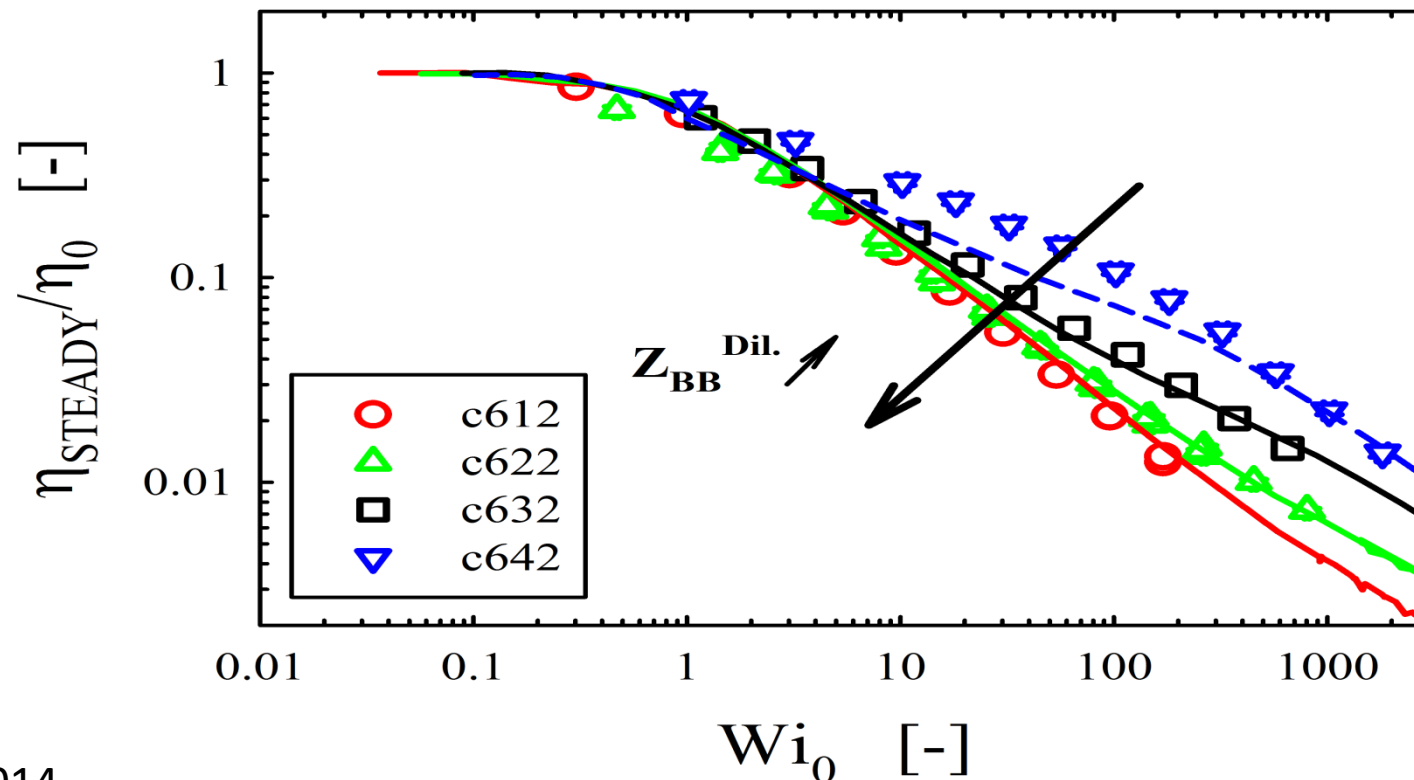
PS 182k: Linear, monodisperse PS
($M_w=182\text{kg/mol}$, $\text{PDI} < 1.1$)

N2380: Linear, polydisperse PS
($M_w=275\text{kg/mol}$, $\text{PDI} < 2.1$)

N2987: Linear, polydisperse PS
($M_w=124\text{kg/mol}$, $\text{PDI} < 1.9$)⁴⁰

SAMPLE	M_{TOT}	M_{BB}	M_{Br}	q
PS c612	477	275	6.5	31
PS c622	626	275	11.7	30
PS c632	918	275	25.7	25
PS c642	1640	275	47	29

Observation 4: Like polydispersity, also branching decreases thinning (and leads to failure of Cox-Merz rule).



Observation 5: Polymers exhibit normal forces

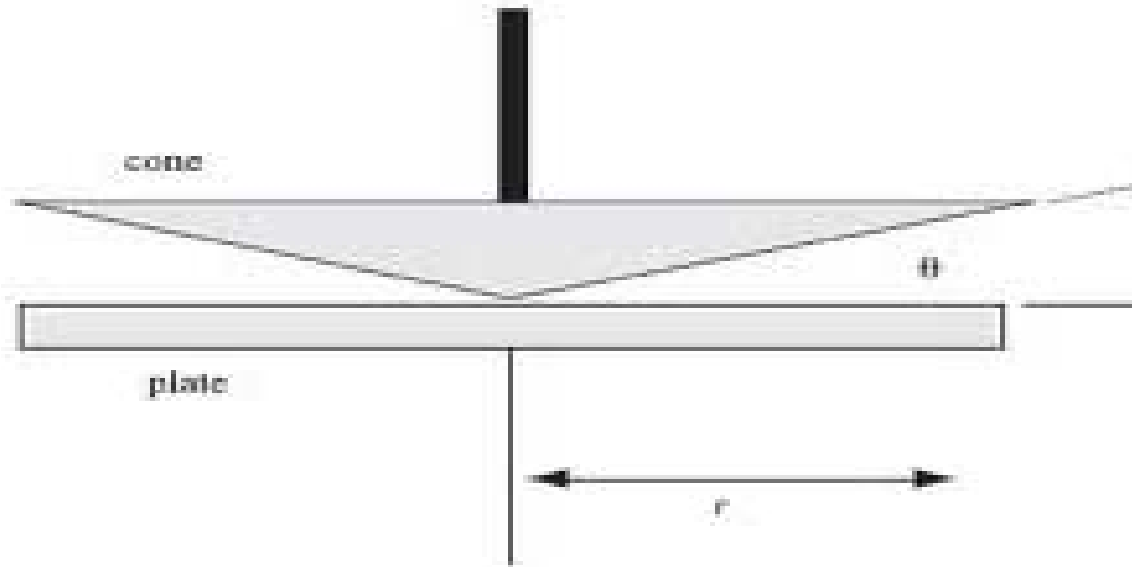
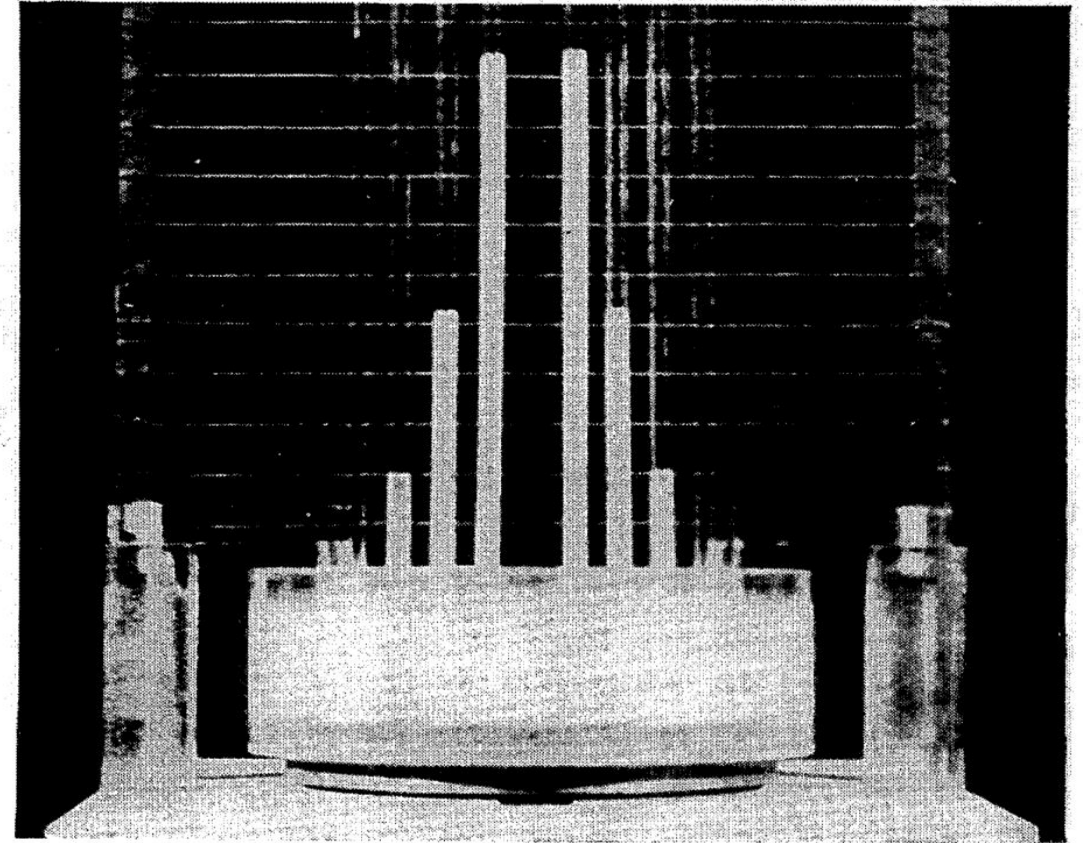
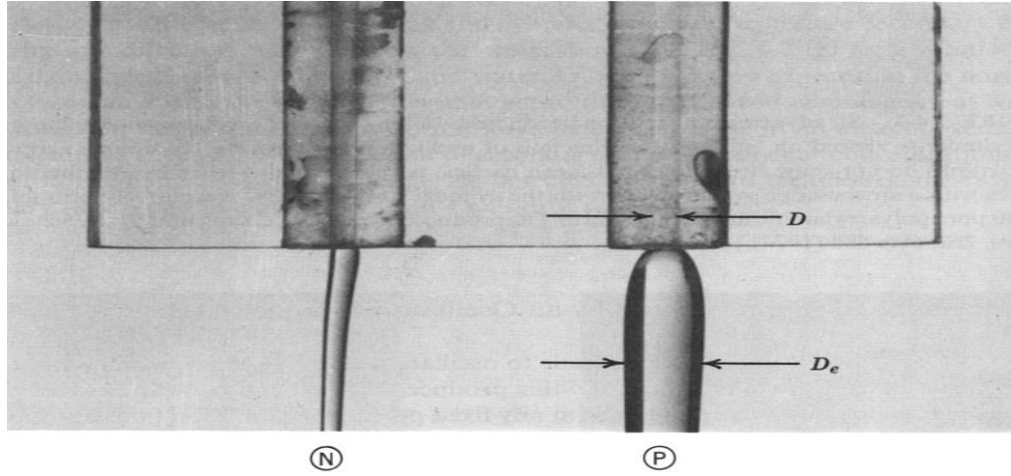


Figure 2. Schematic diagram of the cone and plate rheometer. θ , is angle between cone and plate and r , is the radius of the cone (and well).

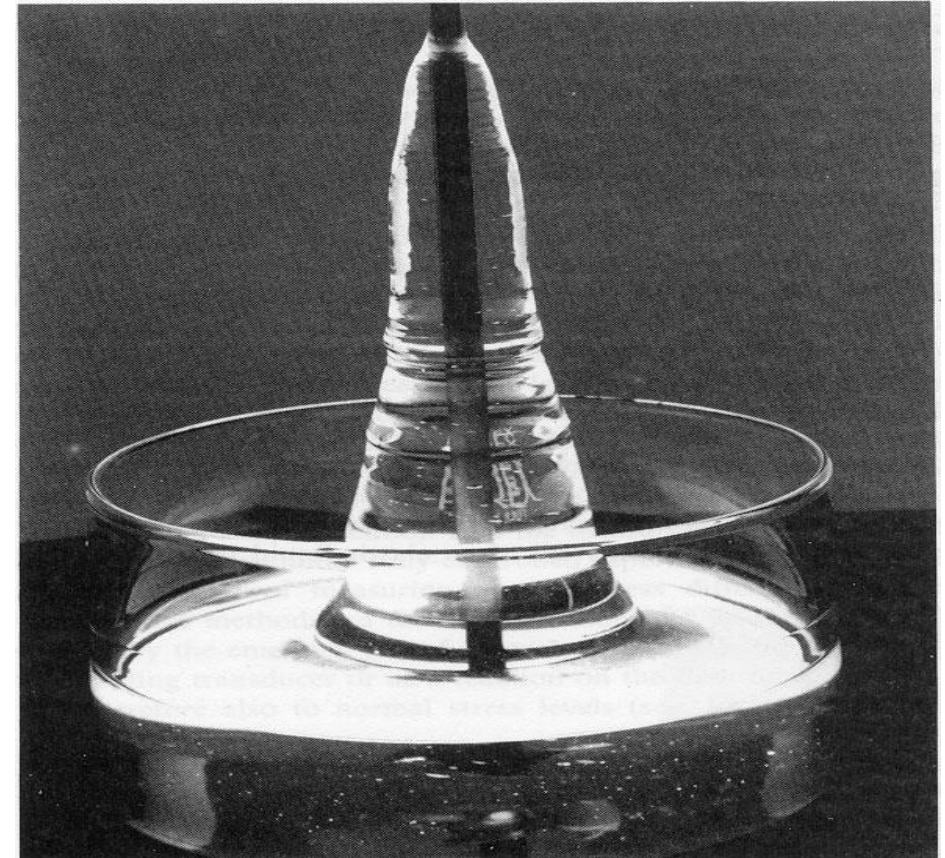


Karl Weissenberg, 1945

Extrudate swell



Rod climbing (Weissenberg)



Observation 5: Polymers exhibit normal forces

$$\gamma_1 = \frac{N_1}{g^2}$$

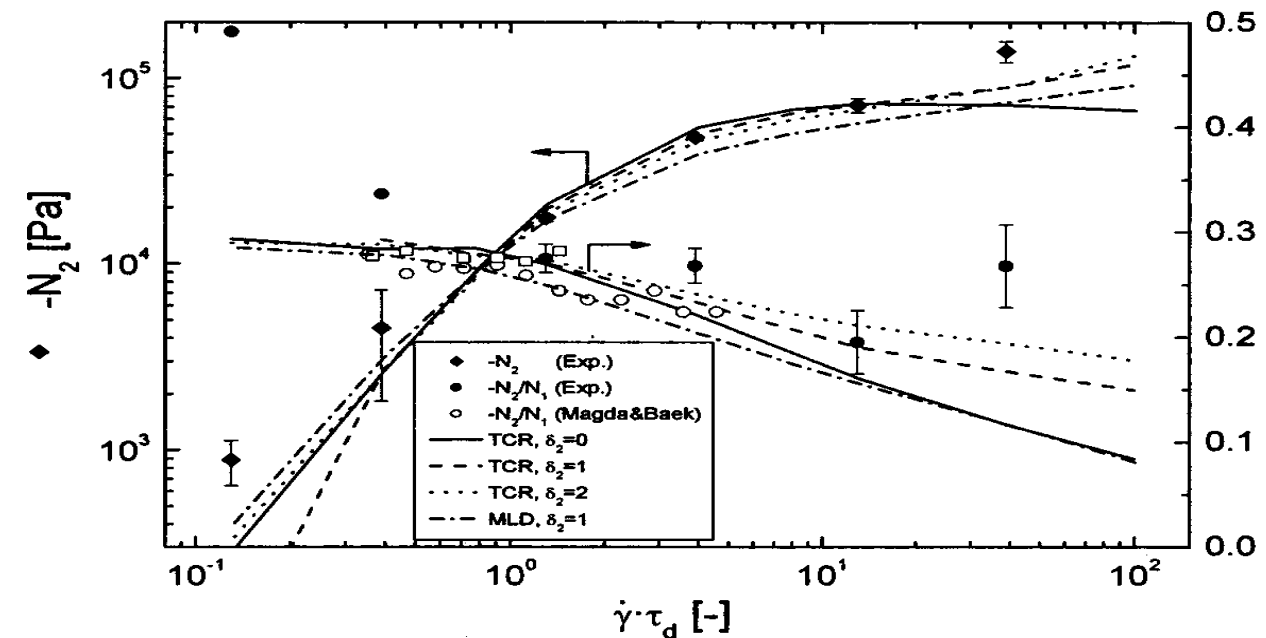
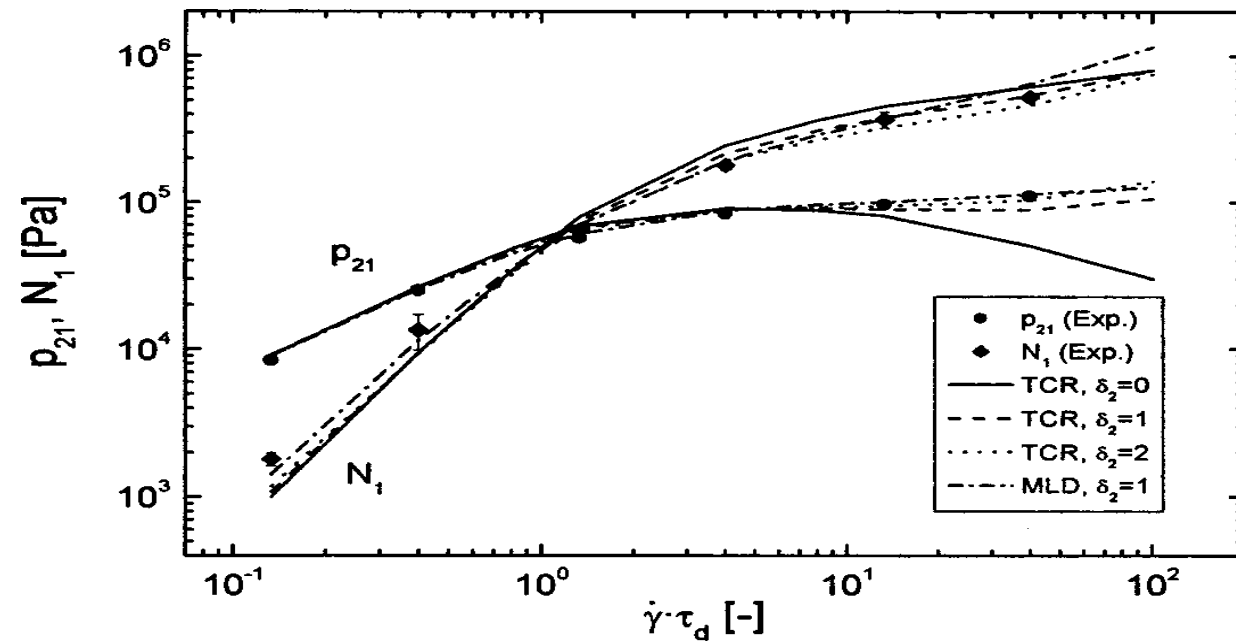
N_1 = First normal stress [Pa]
 γ_1 = First normal stress coefficient [Pa.s²]

Same for N_2

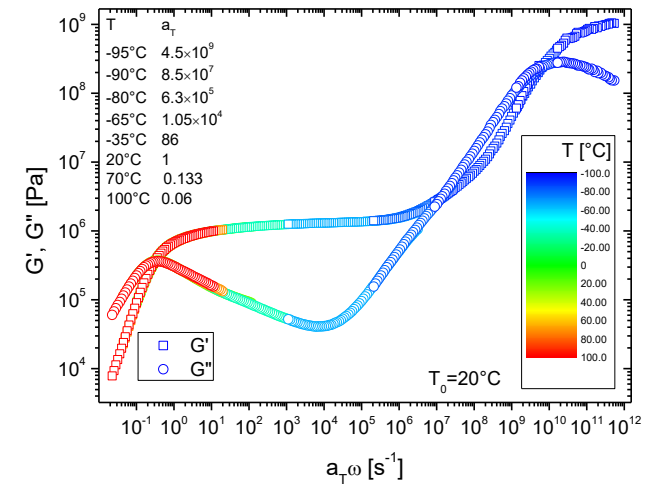
$N_2 < 0$ for polymer melts

$N_2 \ll N_1$

Monodisperse PS 200kg/mol ; Schweizer et al., JOR (2004)



What about monodisperse linear polymers? → Doi-Edwards



1. $\dot{\gamma} < 1/\tau_D$

σ and N_1 increase monotonically to SS

2. $1/\tau_D < \dot{\gamma} < 1/\tau_R$

Max in σ at $\gamma = 2.3$

N_1 monotonic (DE → tube orientation)

3. $1/\tau_R < \dot{\gamma}$

Max in σ shifts to higher γ Asymptote: $\gamma \approx \dot{\gamma} \tau_R$

Max in N_1 appears (DE → chain stretch)

Measure N1 and N2

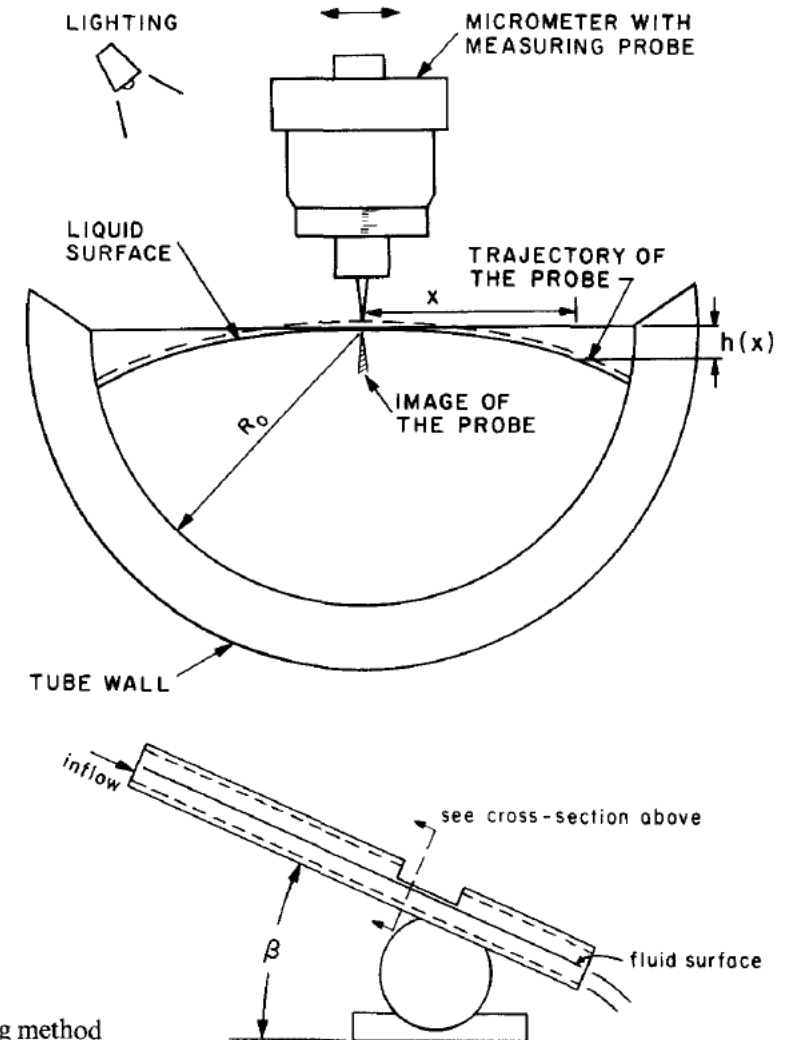
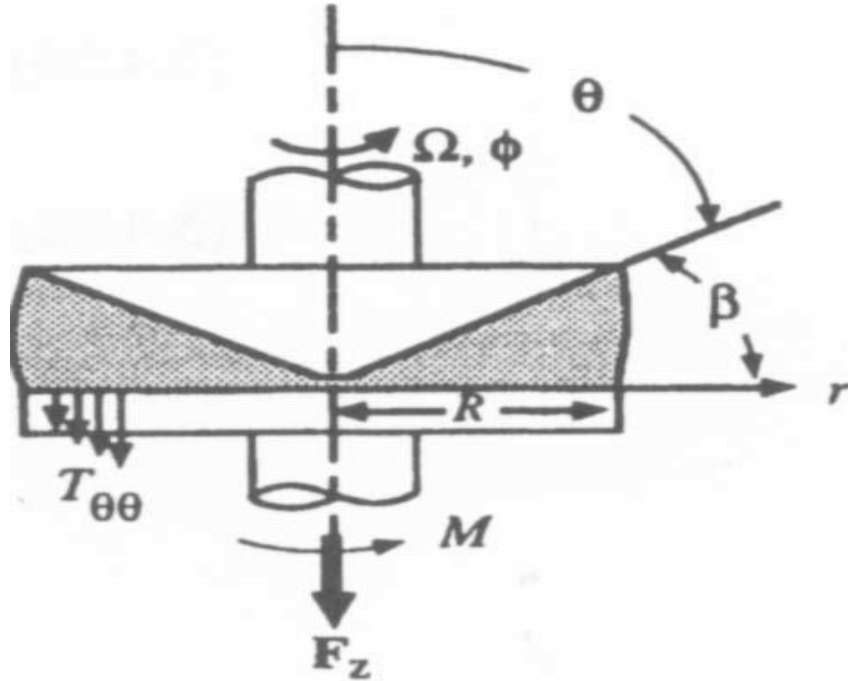
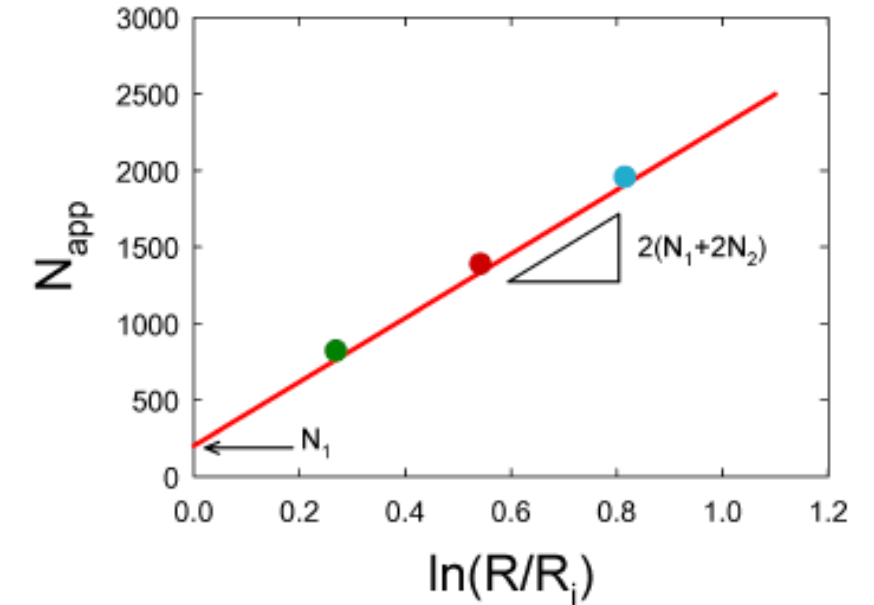
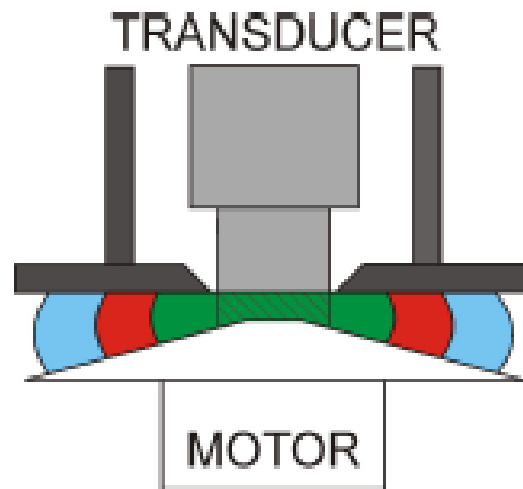
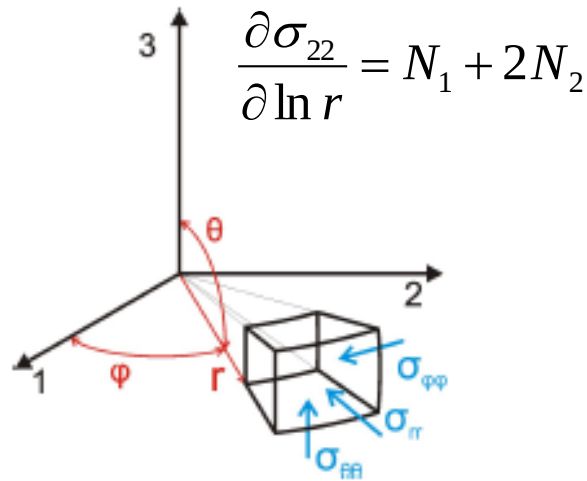


Fig. 1. The channel geometry and the measuring method

N1 and N2 from separate shear measurements with CPP

CPP: initial shape of sample (edge) matters (Schweizer, Denn, Fielding)



$$N_1 = \sigma_{\phi\phi} - \sigma_{\theta\theta}$$

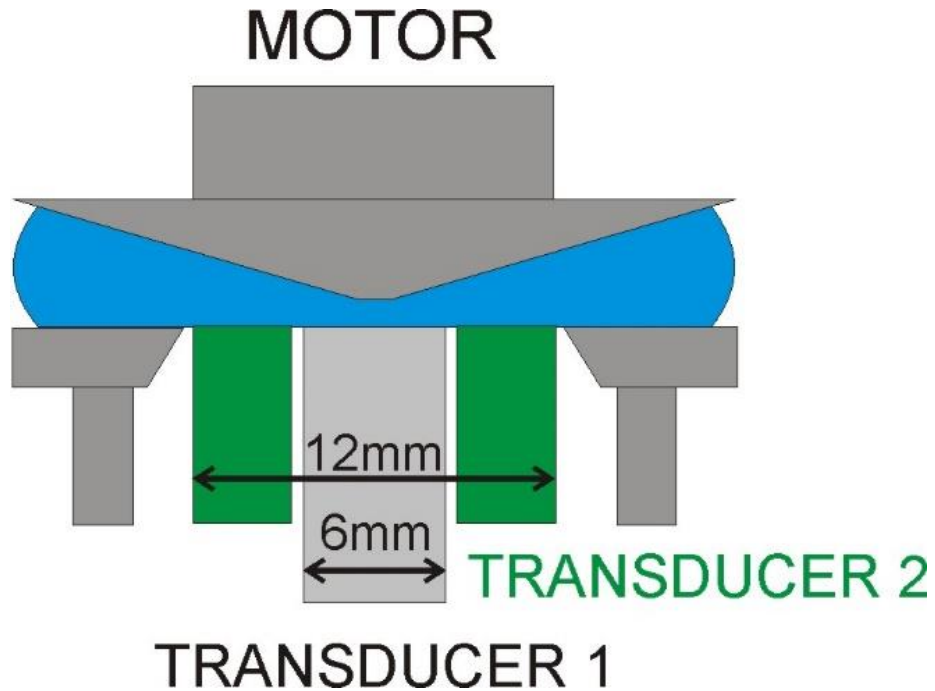
$$N_2 = \sigma_{\theta\theta} - \sigma_{rr}$$

Fixed inner partition/Different external diameters
(n measurements with fixed CPP2)

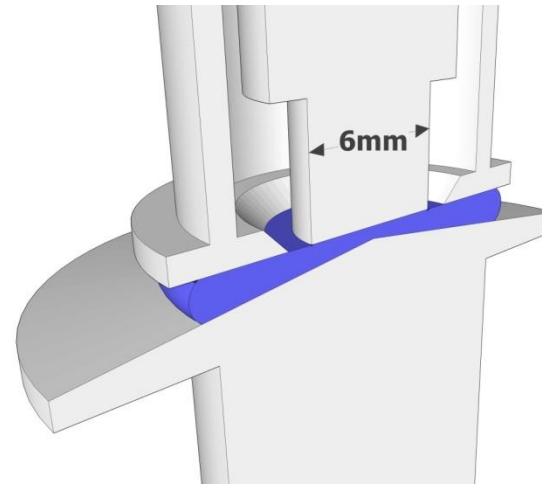
$$N_{app} = \frac{2F_i}{\pi R_i^2} = N_1 + 2(N_1 + 2N_2) \ln \left(\frac{R}{R_i} \right)$$

Alternative (and better) ways to measure N1 and N2

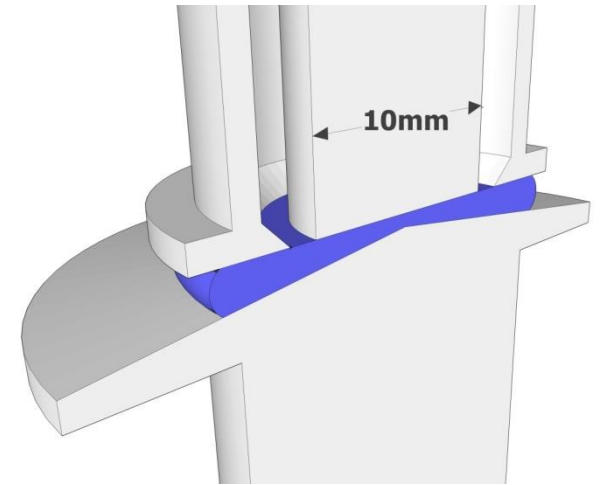
CPP: initial shape of sample (edge) matters (Schweizer, Denn, Fielding)



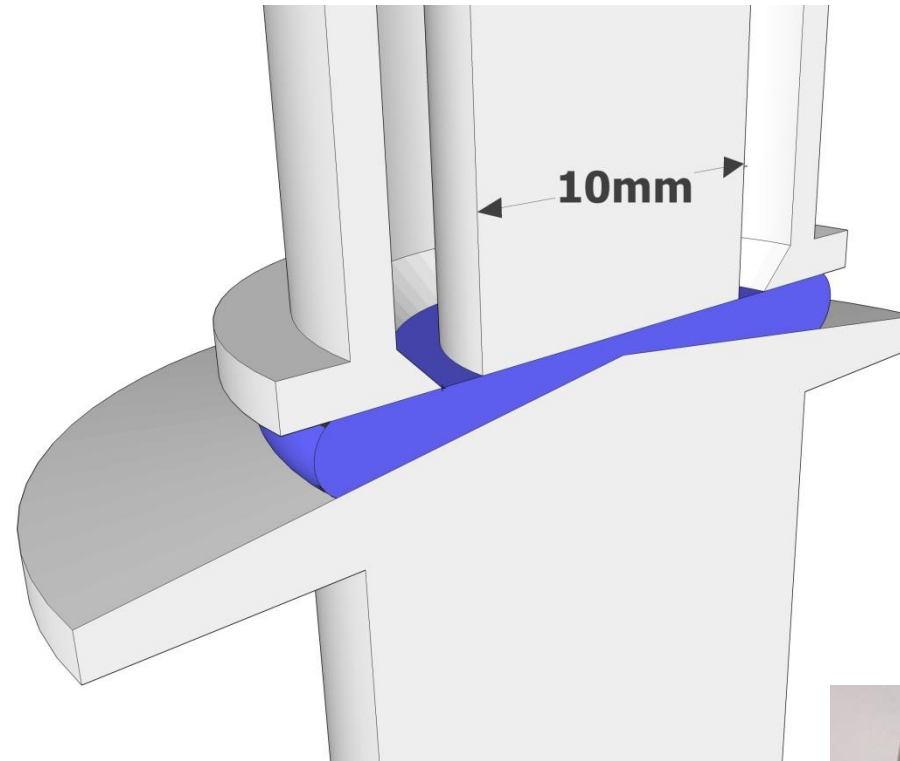
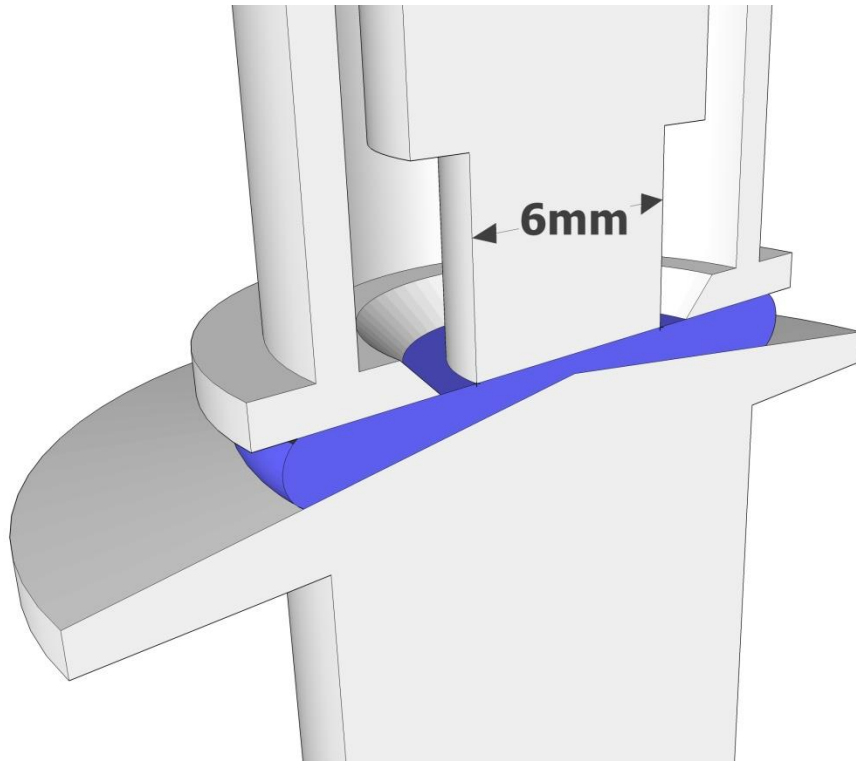
Fixed external diameter/Two different inner radii
(single measurement with double CPP)



Fixed external diameter/Two different inner radii
(two measurements with two CPPs):



Modular CPP: N1 and N2 from two shear measurements

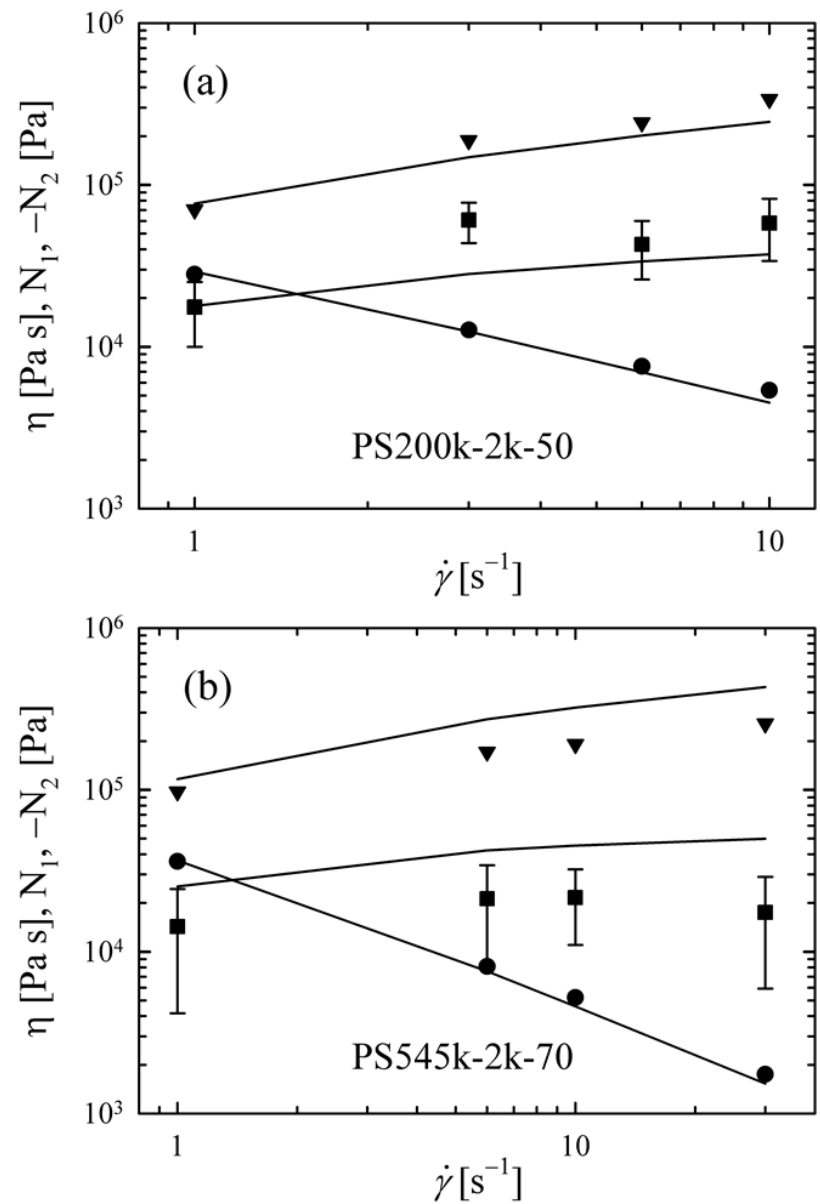


Fixed external diameter/Two different inner radii
(two measurements with CPP2):

$$N_{app} = \frac{2F_1}{\pi R_1^2} = N_1 + 2(N_1 + 2N_2) \ln\left(\frac{R}{R_1}\right)$$



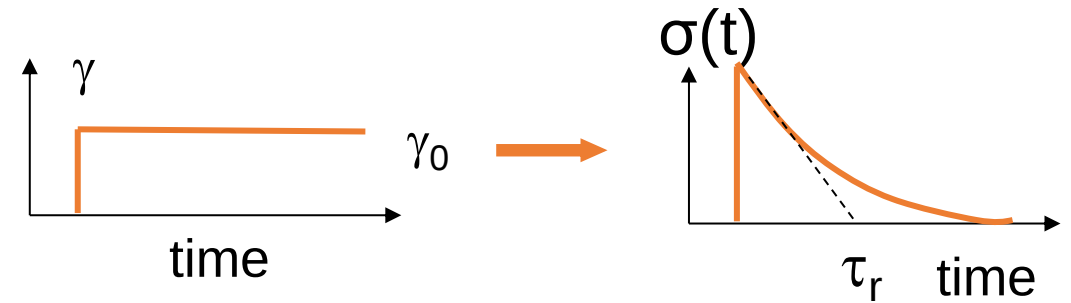
Summary of the results: (a) PS200k-2k-50 ; (b) PS545k-2k-70



$Wi_R = 40$
50

Behavior of polymers in step-strain shear

Sudden imposition of a fixed strain at $t=0$
 Monitor the stress upon relaxation at $t>0$



Cone and plate

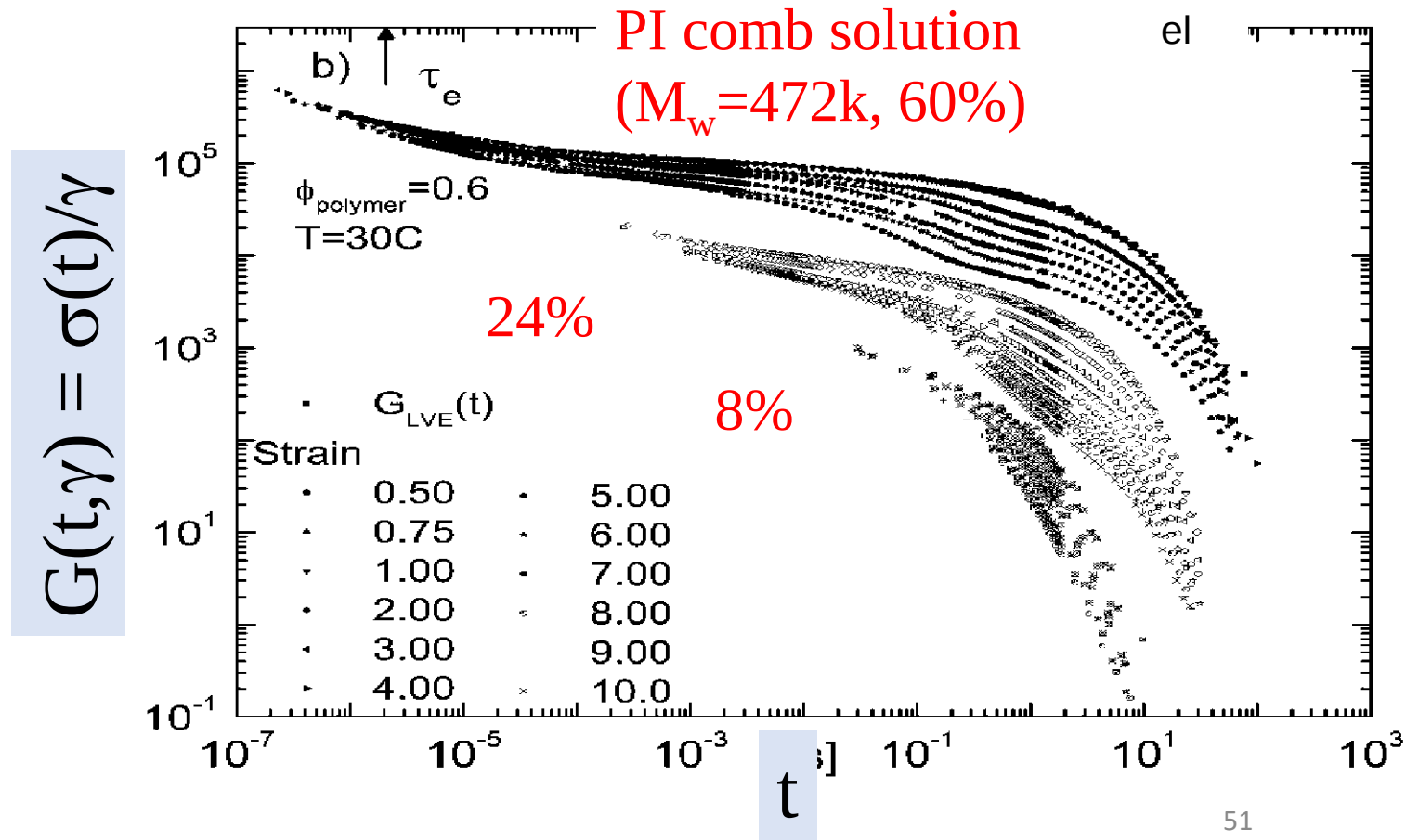
Time-Strain Separability

At long times:

$$G(t, g) = G(t) \cdot h(g)$$

$h(g)$: Damping function

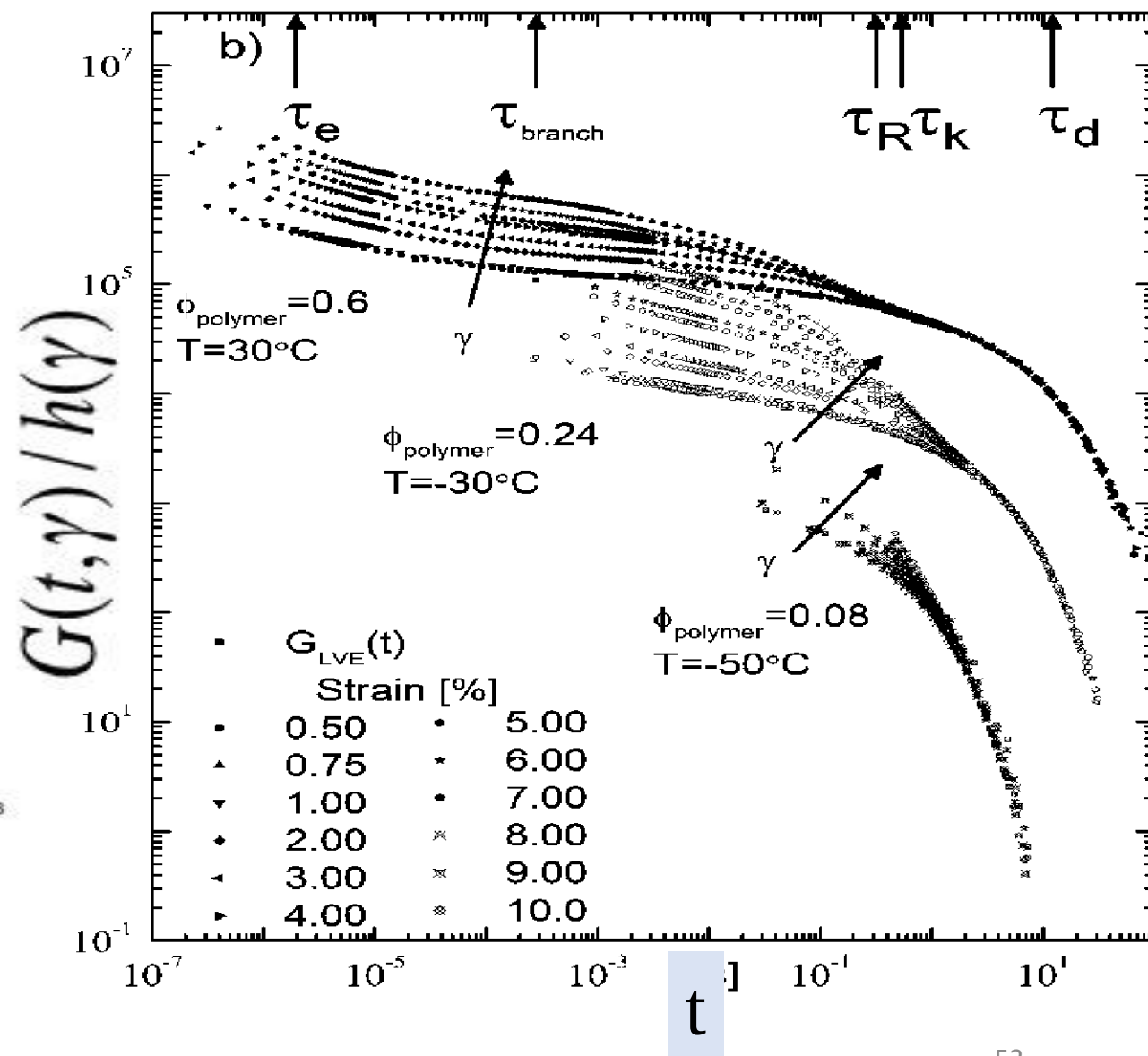
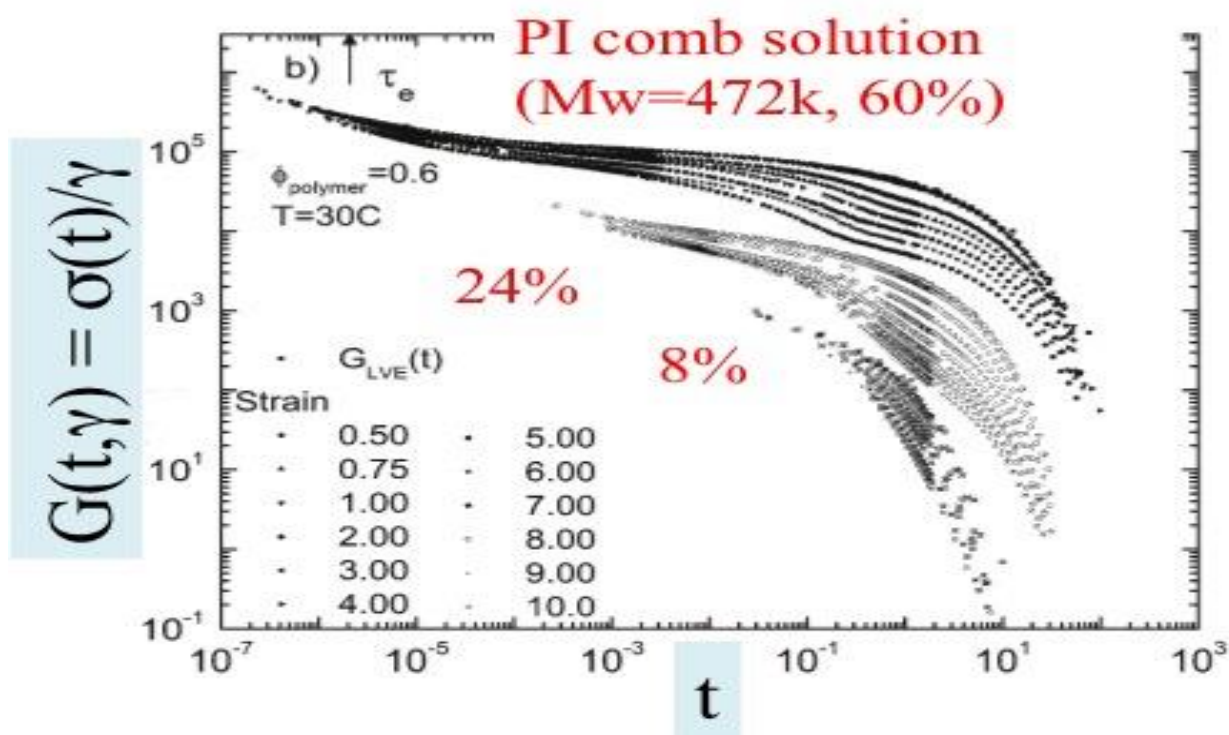
Kirkwood et al., Macromolecules (2009)



At long times:

$$G(t, g) = G(t) \cdot h(g)$$

$h(g)$: Damping function



$$G(t, g) = G(t) \cdot h(g)$$

$h(\gamma)$: Damping function

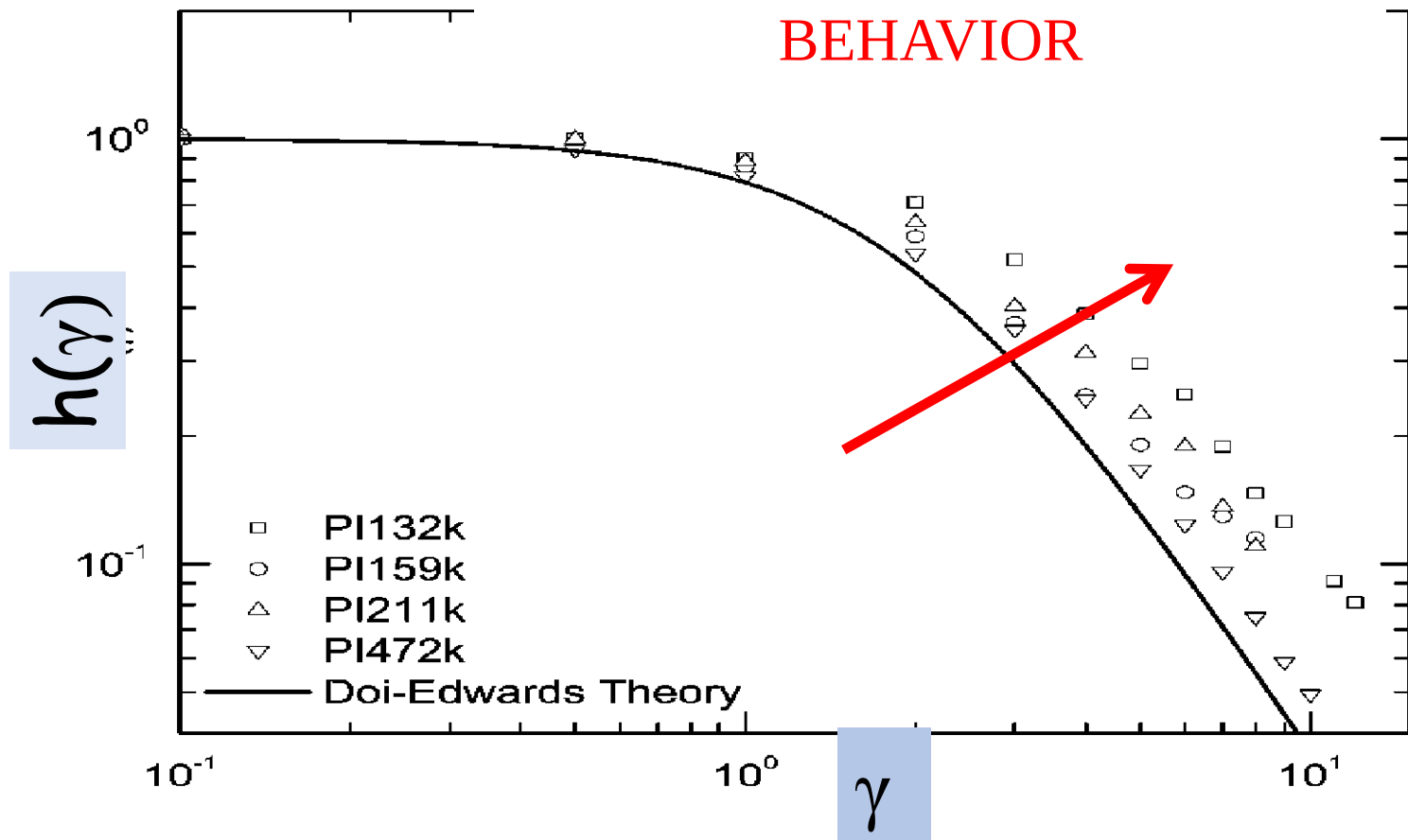
Theory for linear, monodisperse polymers (Doi-Edwards):

$$h(g) = \frac{1}{1 + \frac{4}{15}g^2}$$

Polydisperse linear polymers have identical $h(\gamma)$; IF molar mass of “all components” is high enough

Ye and Sridhar, Macromolecules (2005)

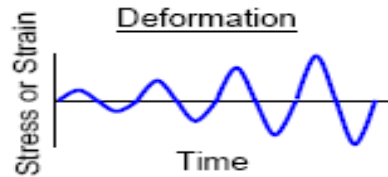
BRANCHING LEADS TO A WEAKER DAMPING BEHAVIOR



Kirkwood et al., Macromolecules (2009)

Large amplitude oscillatory shear (LAOS)

Dynamic Stress or Strain Sweep (Torque Ramp)

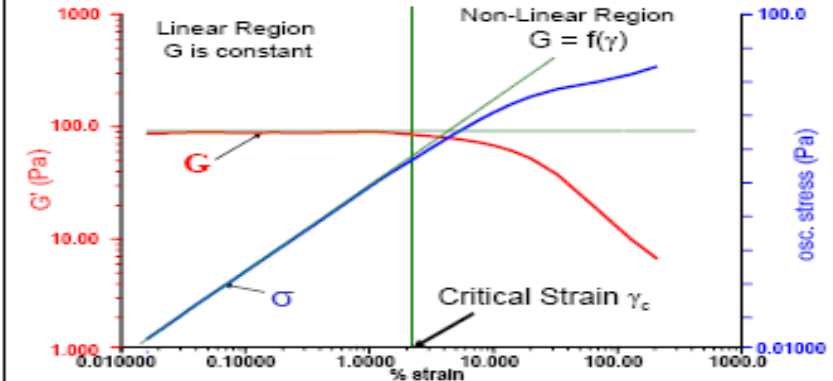


• The material response to increasing deformation amplitude (stress or strain) is monitored at a constant frequency and temperature.

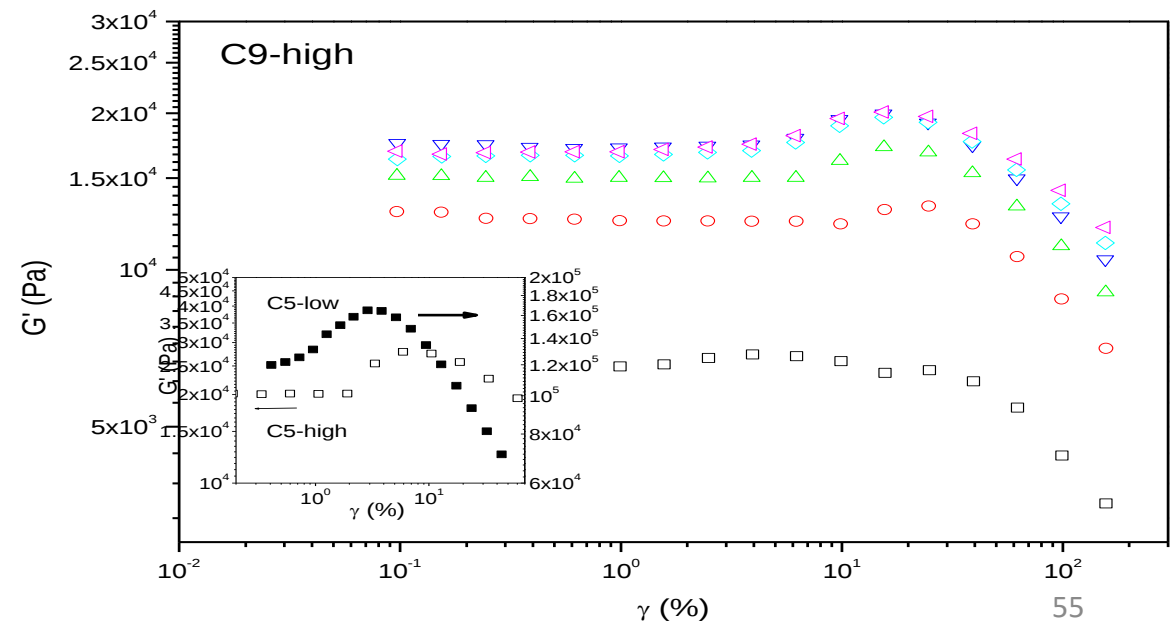
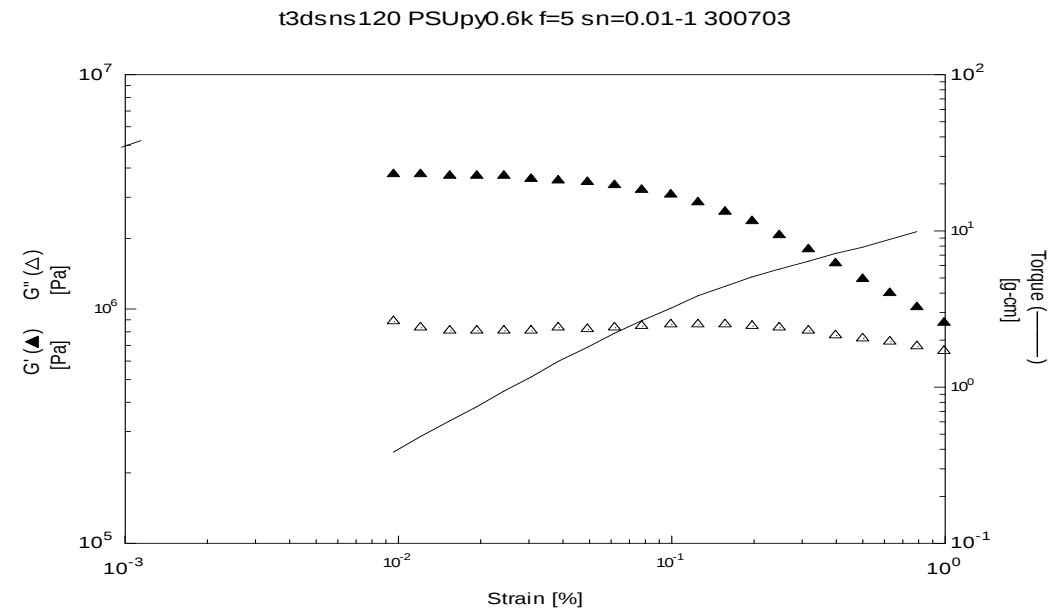
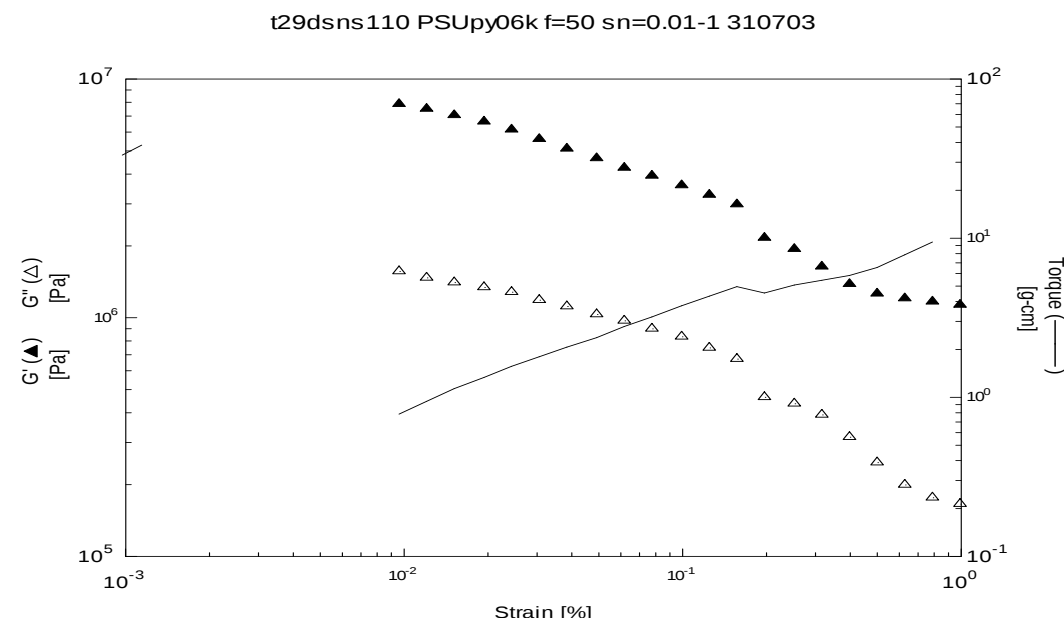
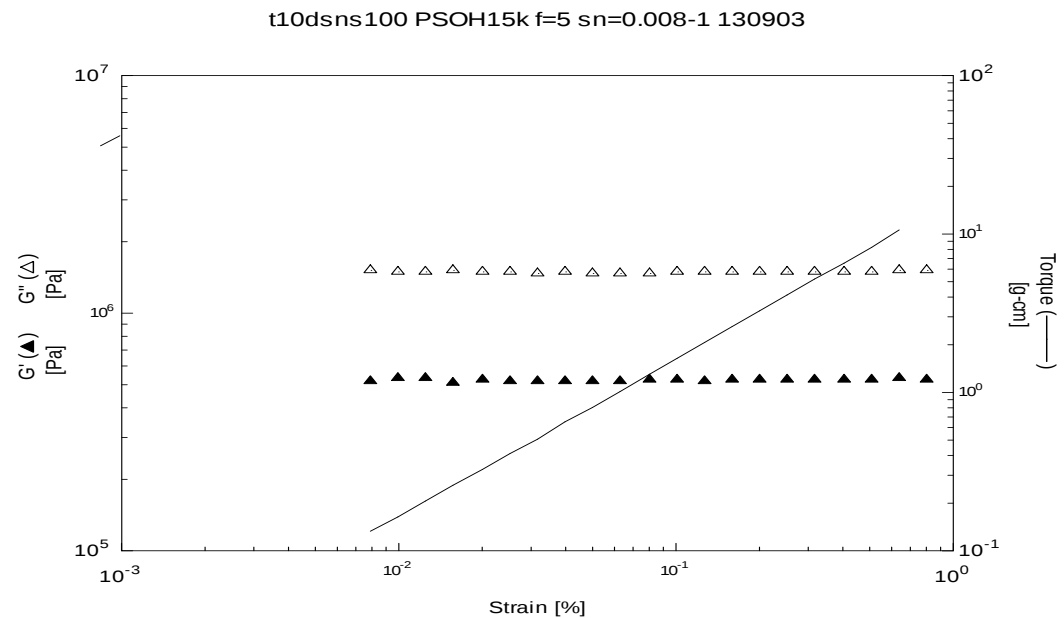
• USES

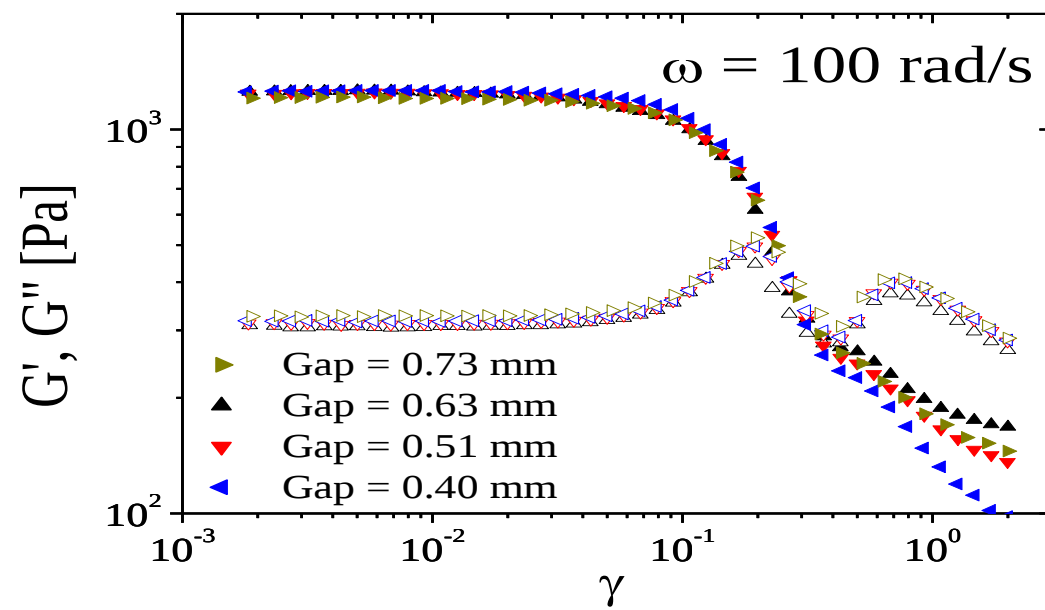
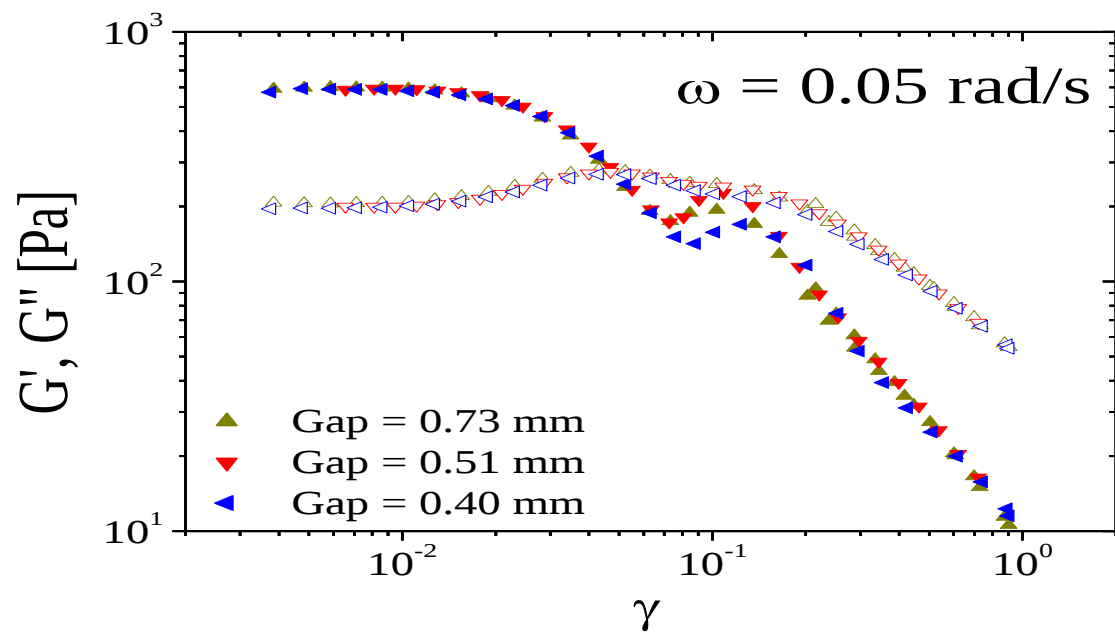
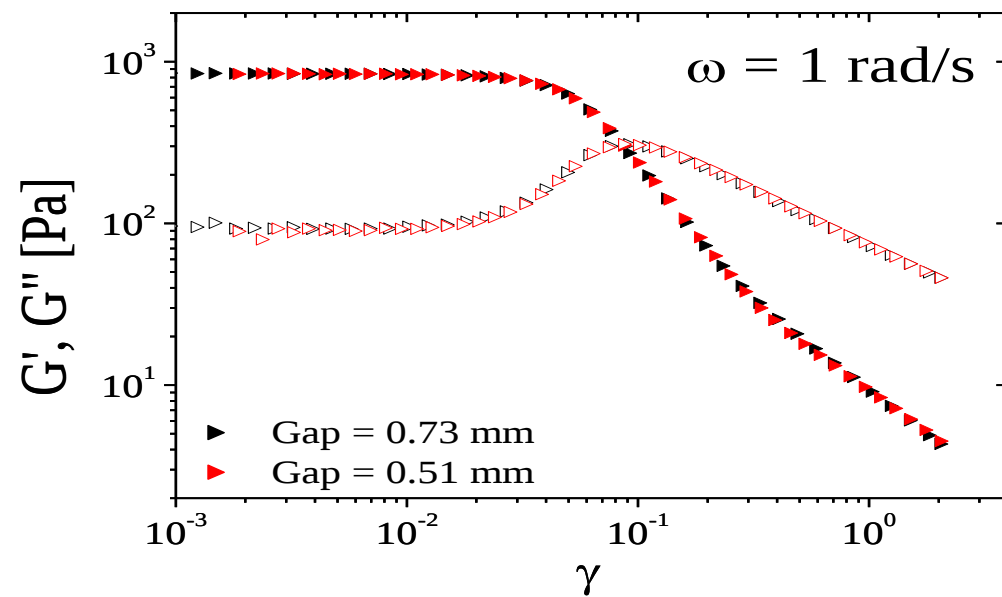
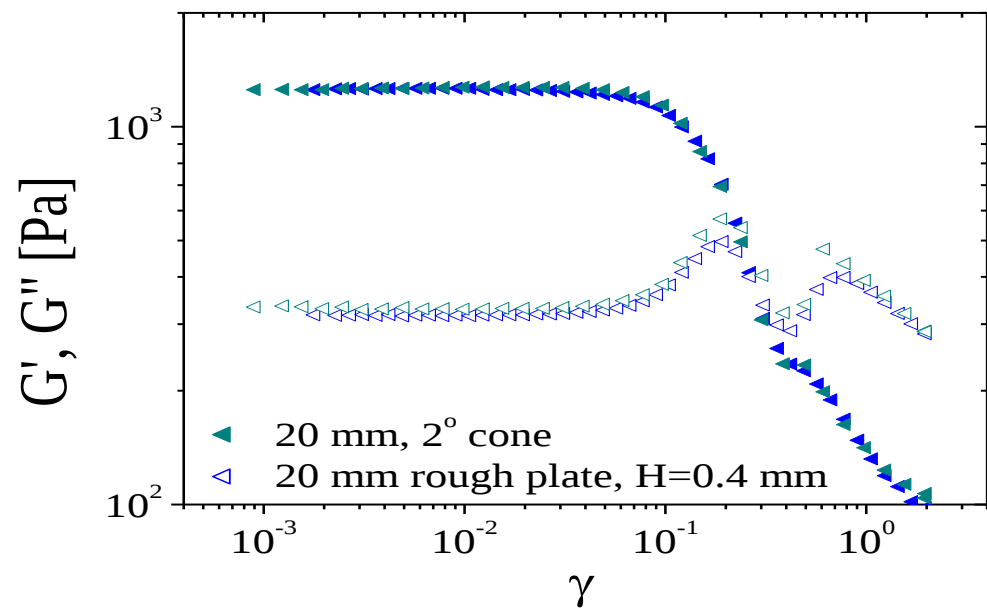
- Identify Linear Viscoelastic Region
- Strength of dispersion structure - settling stability
- Resilience

Dynamic Strain Sweep: Material Response



Linear and SCLC polymers



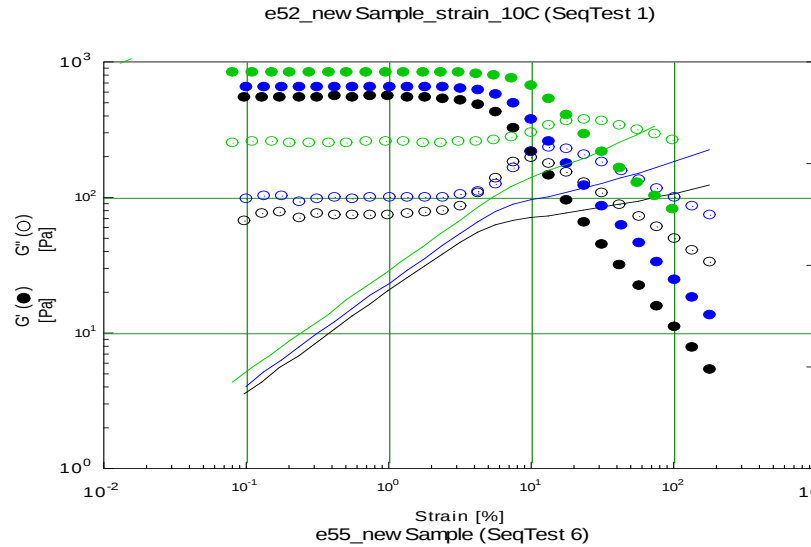


Tunable micelles, E. van Ruymbeke et al., 2007

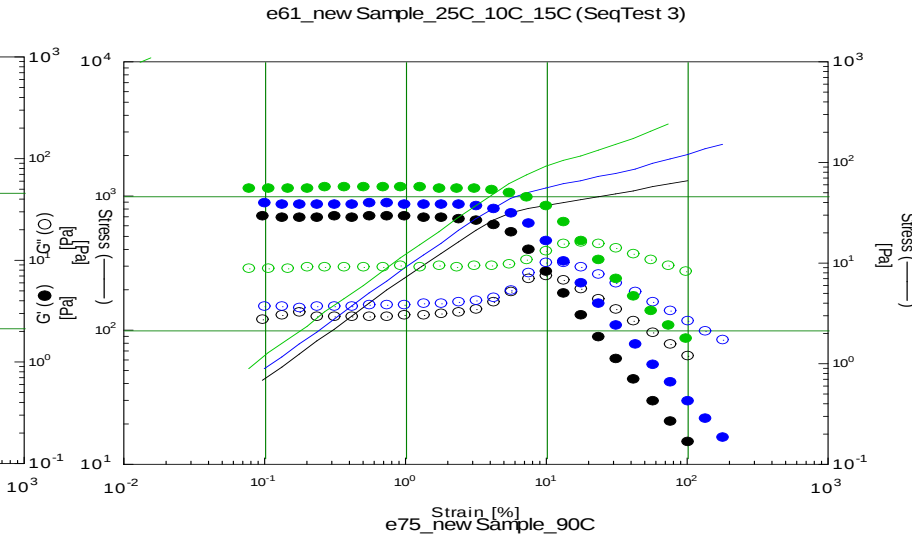
1 rad/s, 10 rad/s or 100 rad/s

2c*:

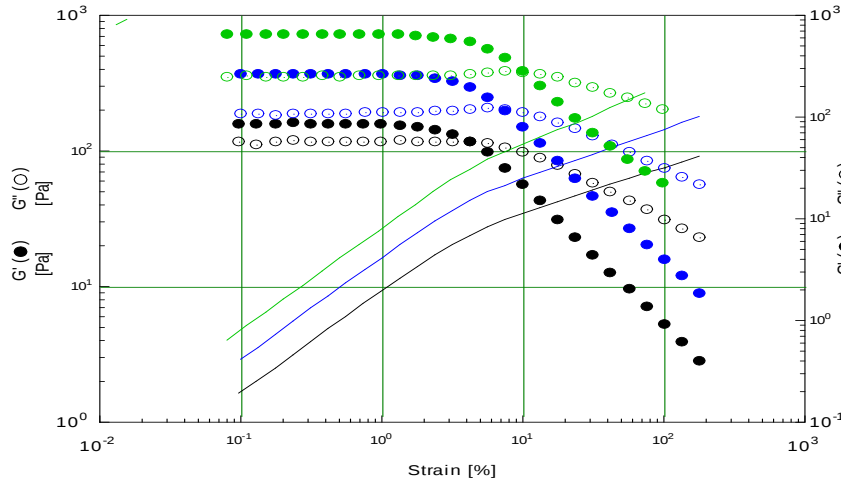
10C



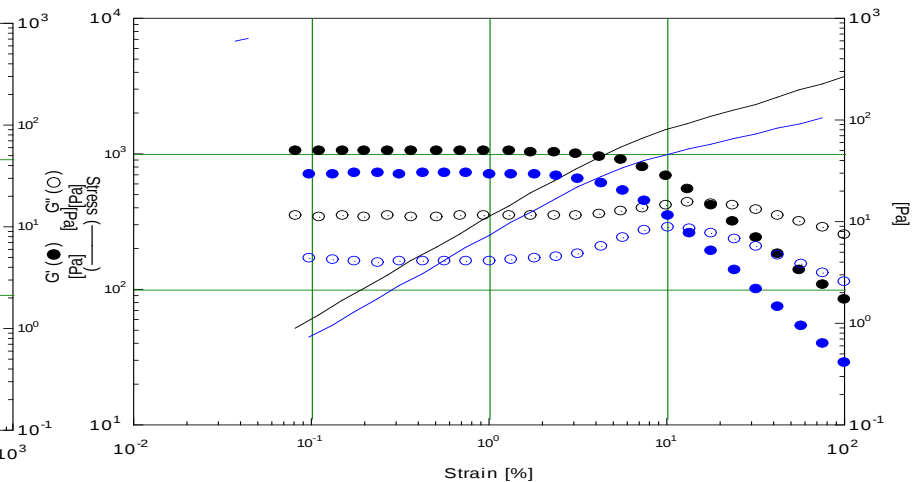
25C



50C

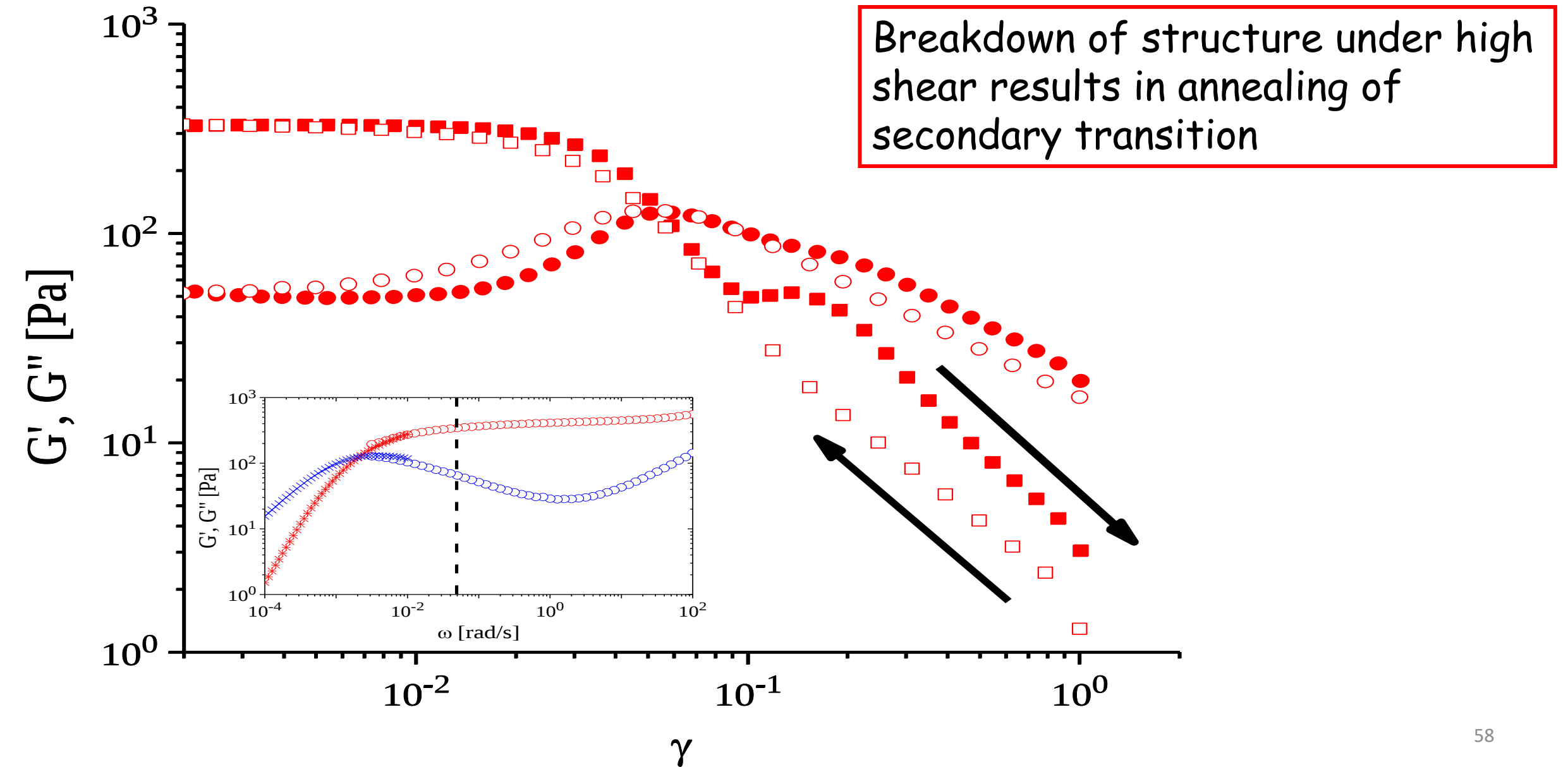


90C

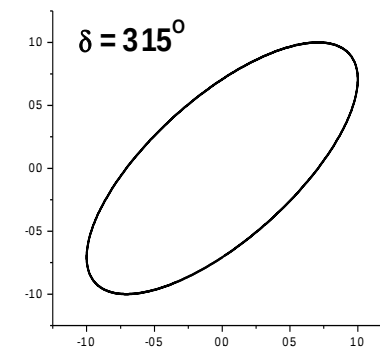
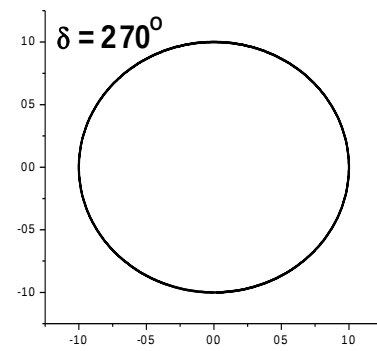
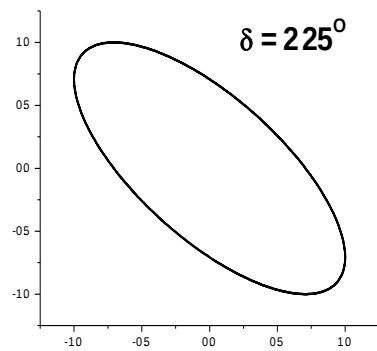
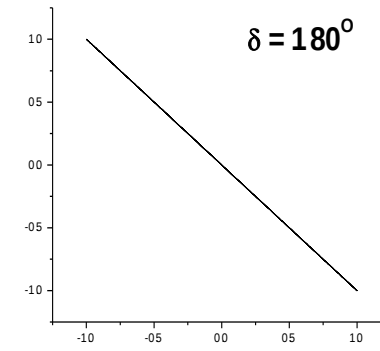
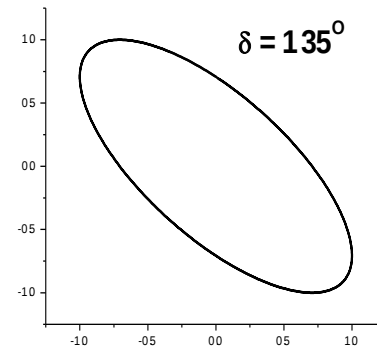
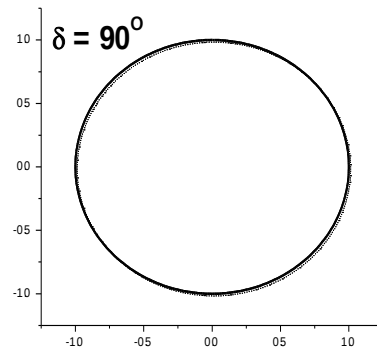
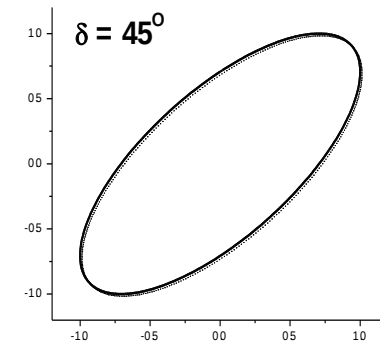
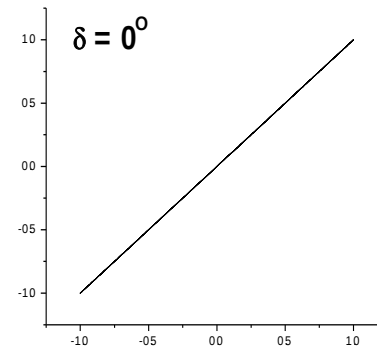
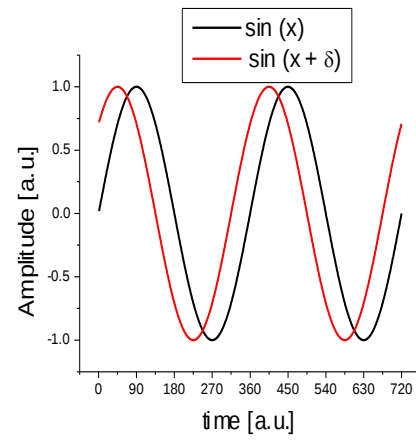


10 and 100 rad/s

Thixotropic behavior



Lissajous-Bowditch plots



Steady vs oscillatory shear:

Which one is a better (more straightforward)
probe of (i) LVE and (ii) NLVE ?

LAOS and MAOS with polymers (and other soft matter systems):

Dealy, Giacomini
(sliding plate rheometry)

Recent advances:

R. Ewoldt, S. Rogers, Univ. of Illinois @ Urbana-Champaign