A close look at the rim of viscous sheets

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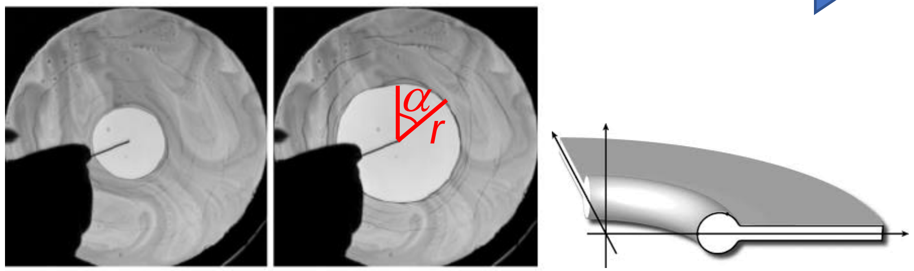
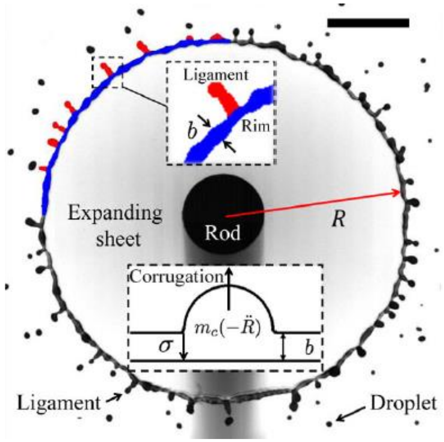
Context – free liquid sheet

At the edges of a free liquid sheet

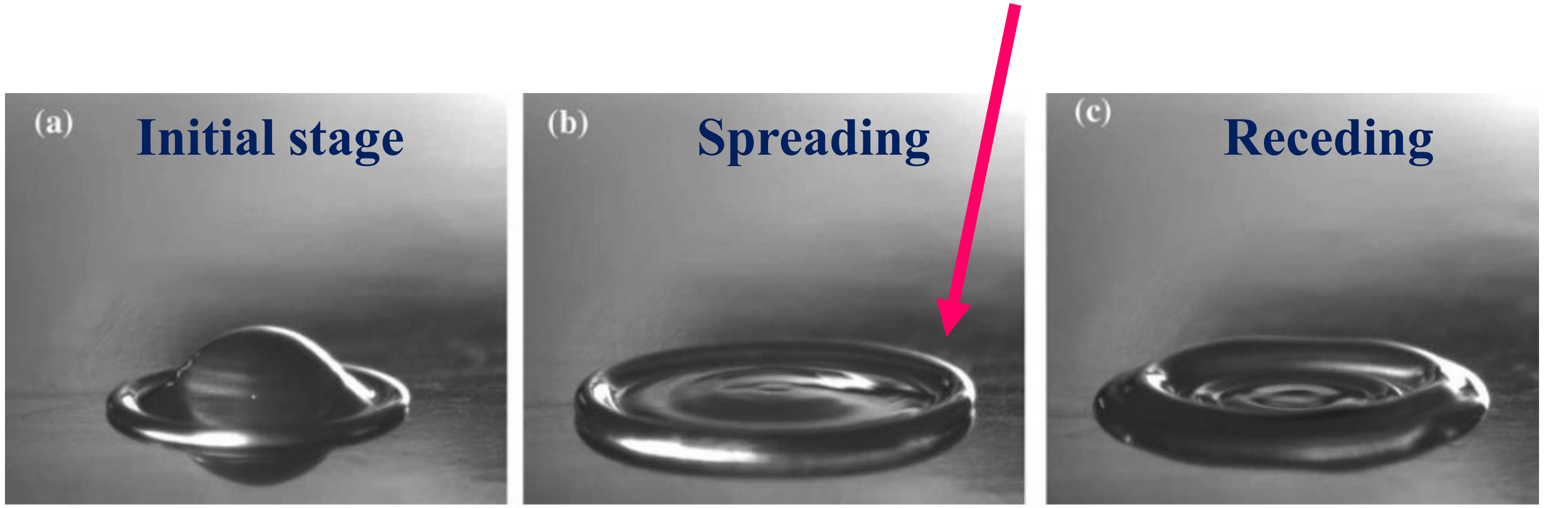
Emergence and growth of a rim



Culick. *Journal of Applied Physics* (1960)
Taylor. *Proceedings of the Royal Society of London* (1959)



Wang et al. *Phys. Rev. Letters* 120, 204503 (2018)



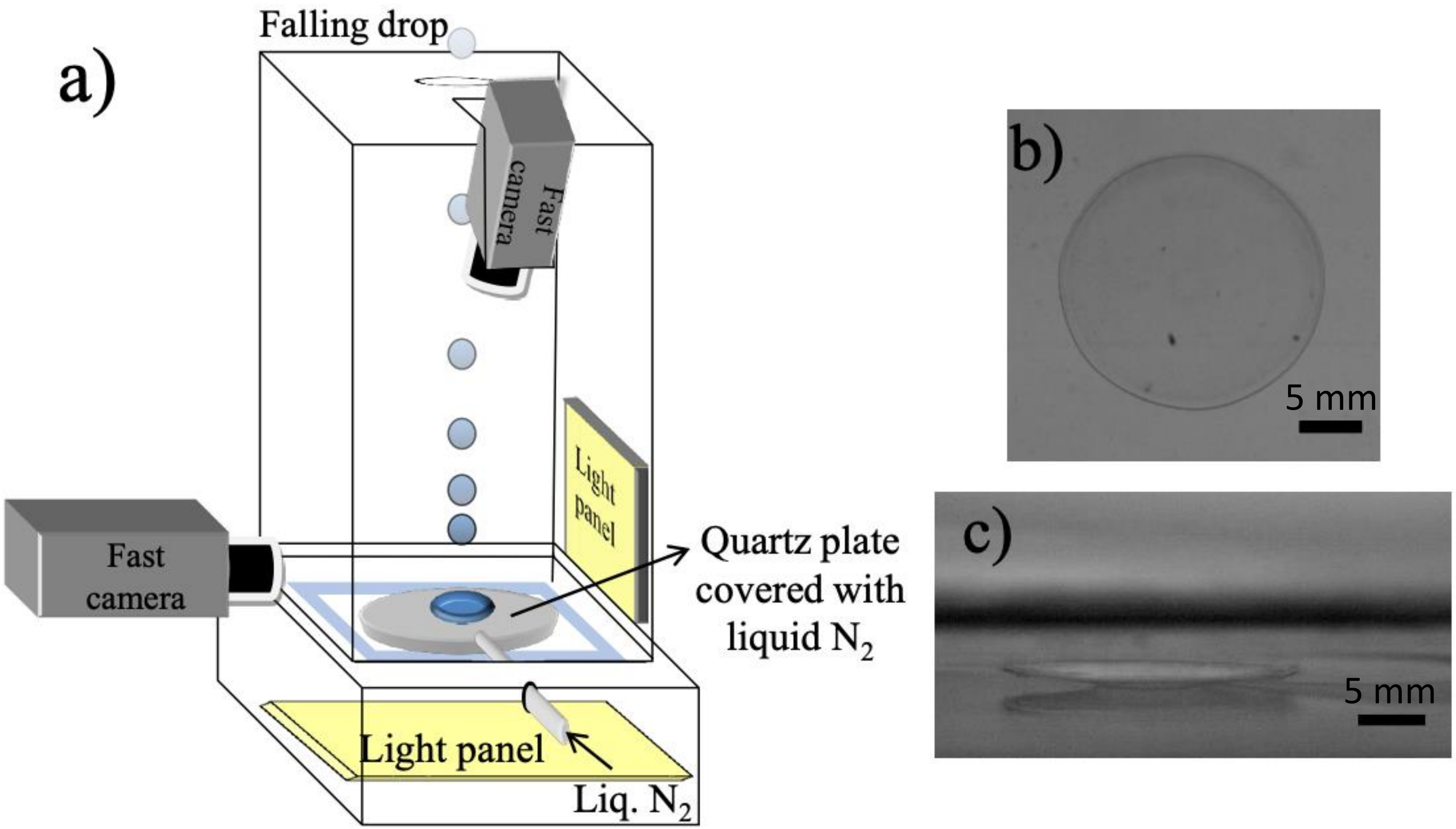
Stages of drop impact onto a dry, rigid, smooth, partially wettable substrate (Roisman et al.2002).

Very few experimental analysis of the growth dynamics of a rim during expansion of a liquid sheet

Experimental set-up

$$v_0 = 4.2\mathrm{ms}^{-1}$$

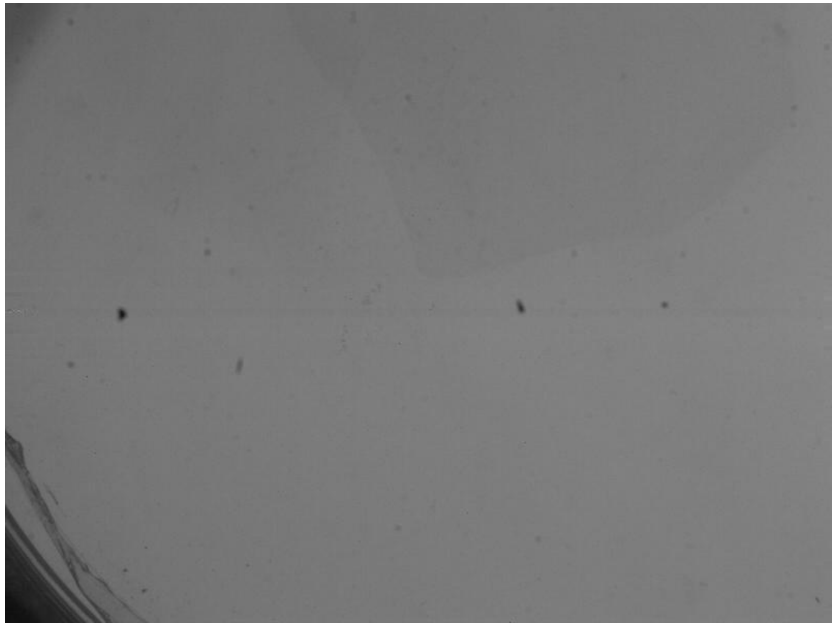
$$d_0 = 4\mathrm{mm}$$



Glycerol/water mixtures $60\mathrm{mPa.s} \leqslant \eta \leqslant 813\mathrm{mPa.s}$ $\gamma = 65\mathrm{mNm}^{-1}$

Drop impact

83% Glycerol – water
η =76 mPa s

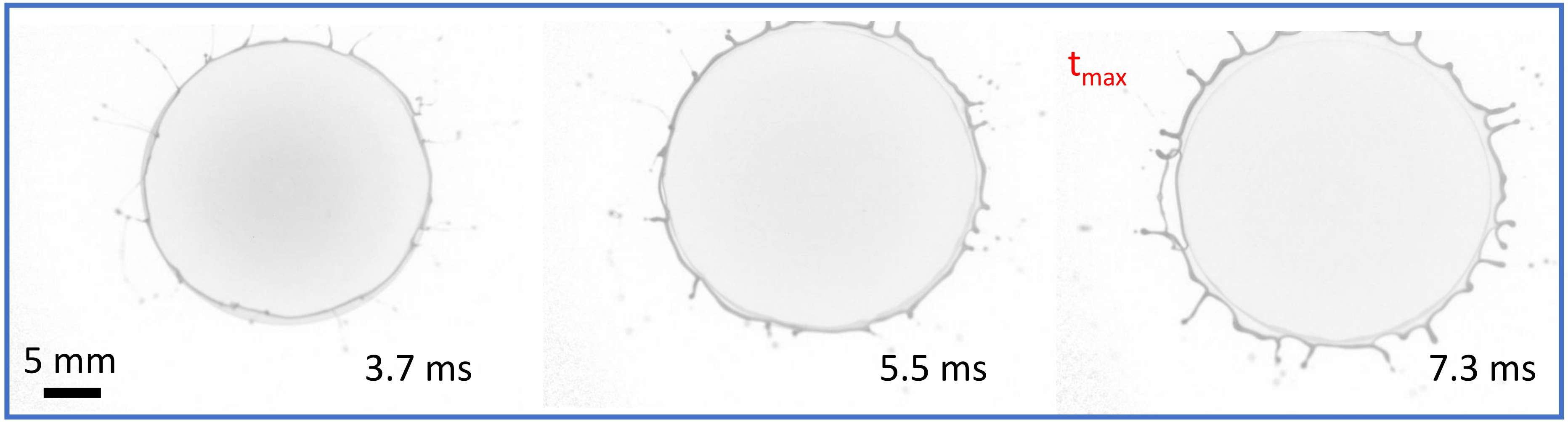


97.5% Glycerol – water
η = 813 mPa s

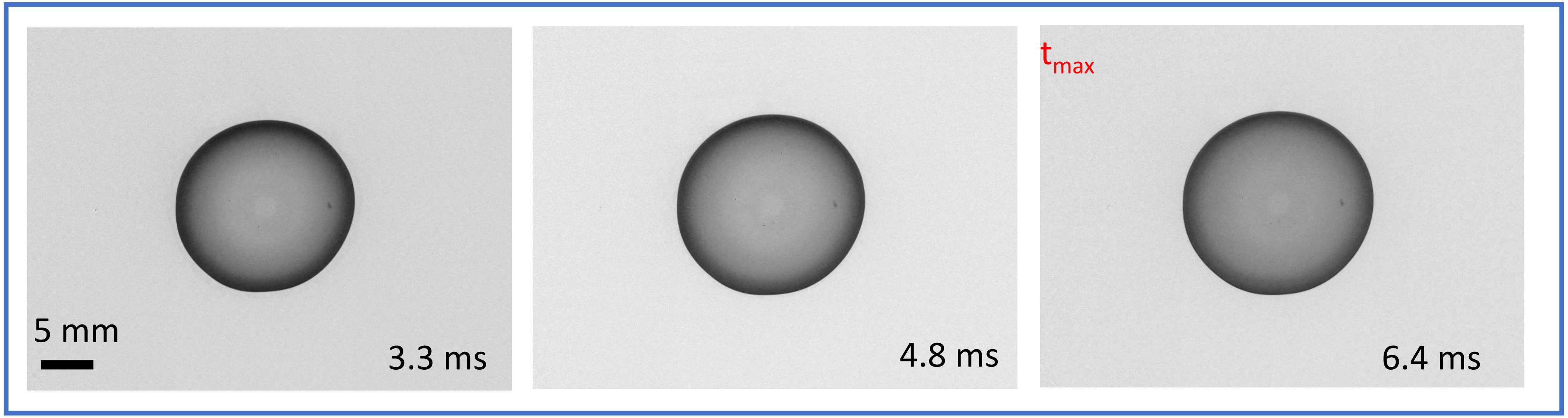
$v_0 = 4.2\mathrm{ms}^{-1}$

Sheet thickness field

Water – Glycerol 80% $\eta = 60$ mPas



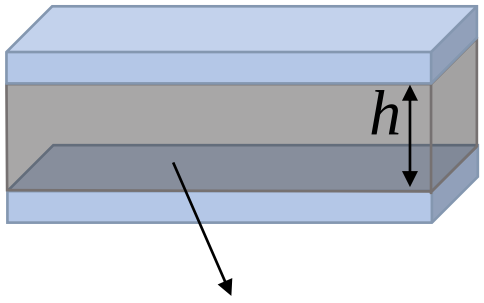
Water – Glycerol 97.5% $\eta = 813$ mPas



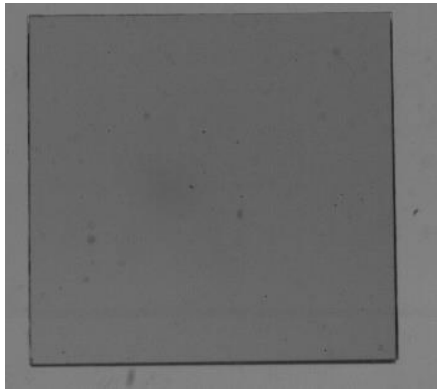
Sheet thickness field

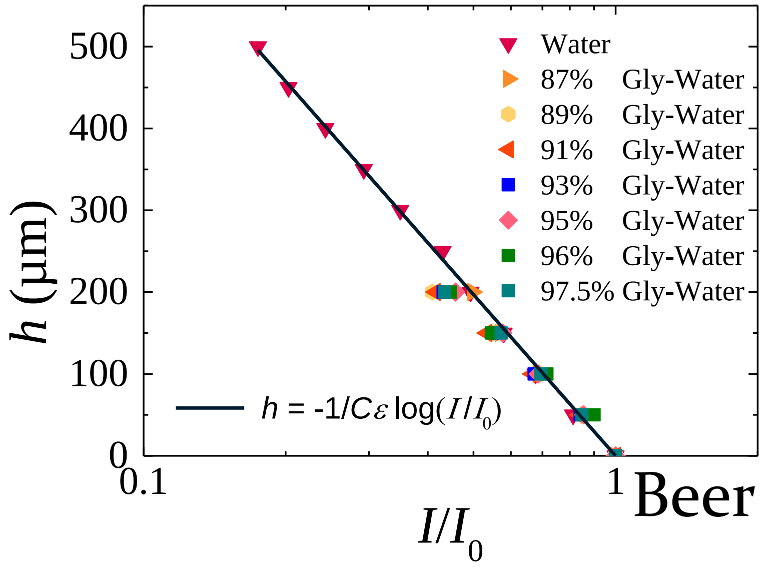
Calibration

Grey level of the image ⟺ **Thickness** of the sheet

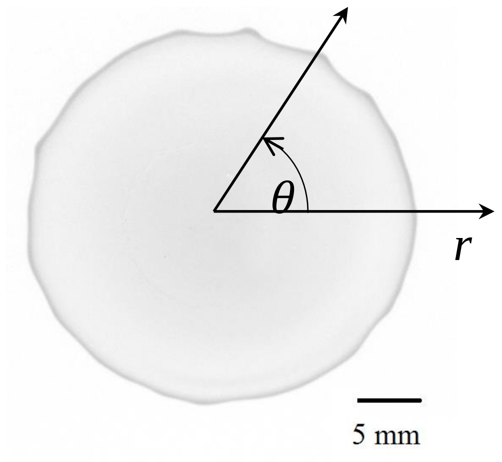


Gly/water +
Nigrosine (dye) at 1.19g/L



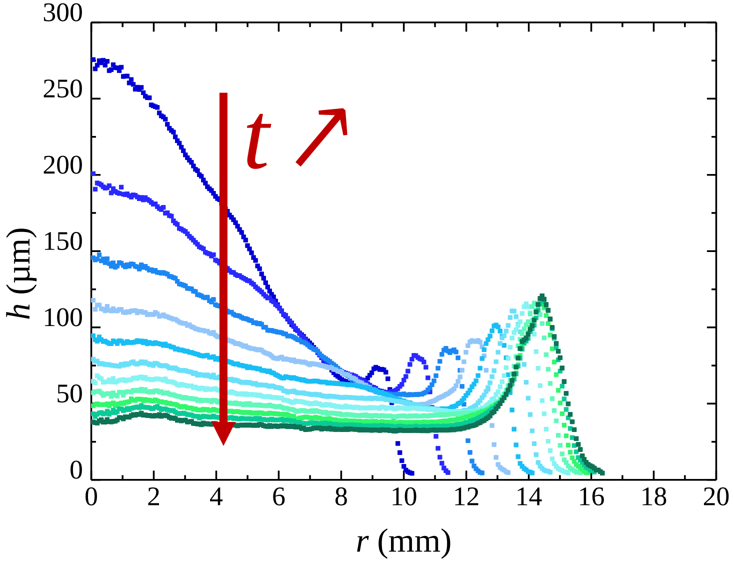


Beer Lambert



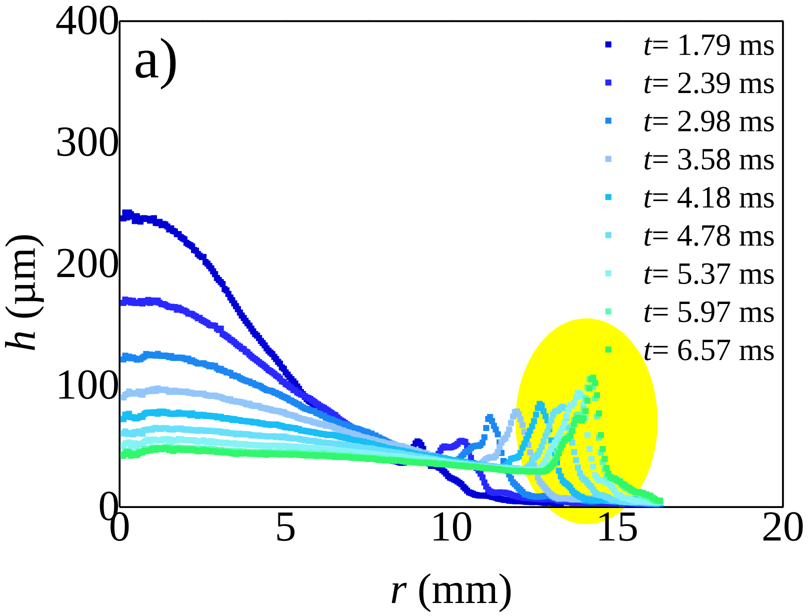
Azimuthal averaging (axial symmetry)

$$< h(r,\theta,t) >_\theta = h(r,t)$$

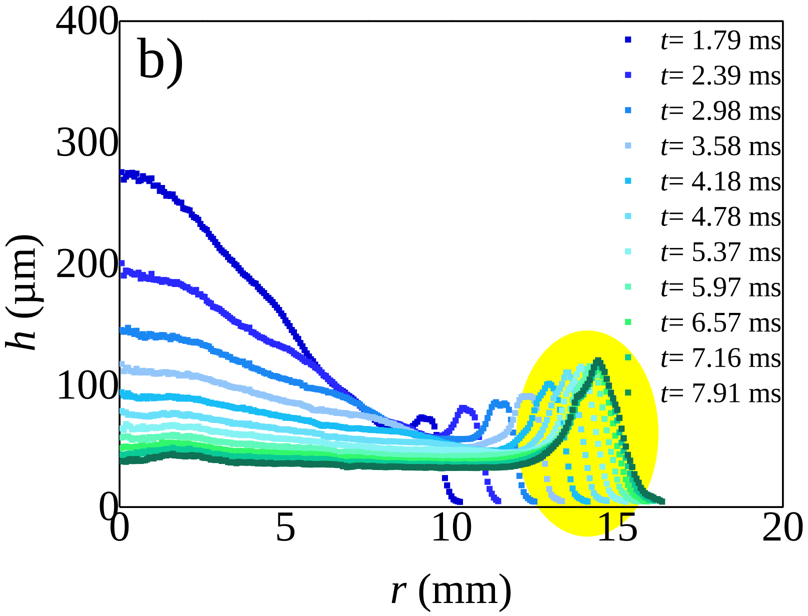


Sheet thickness field

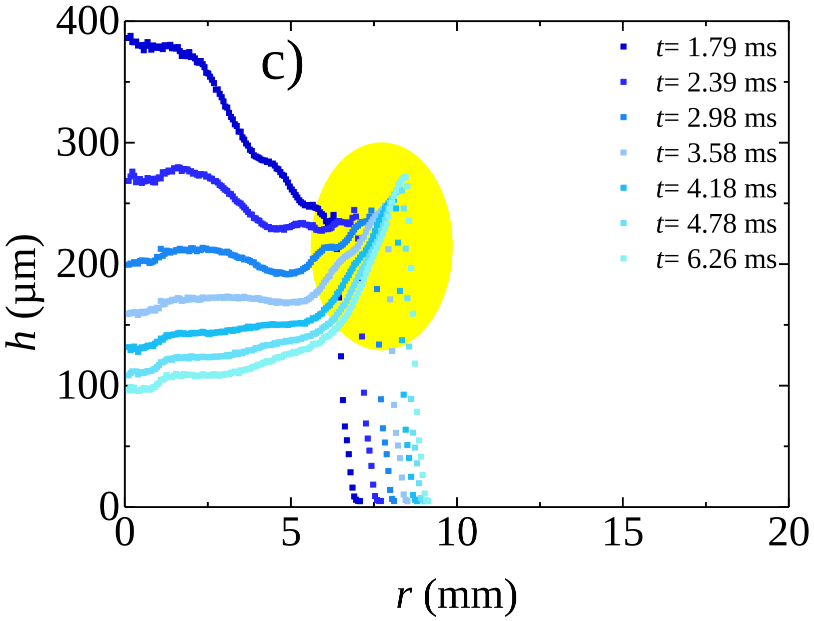
$\eta = 60$ mPas



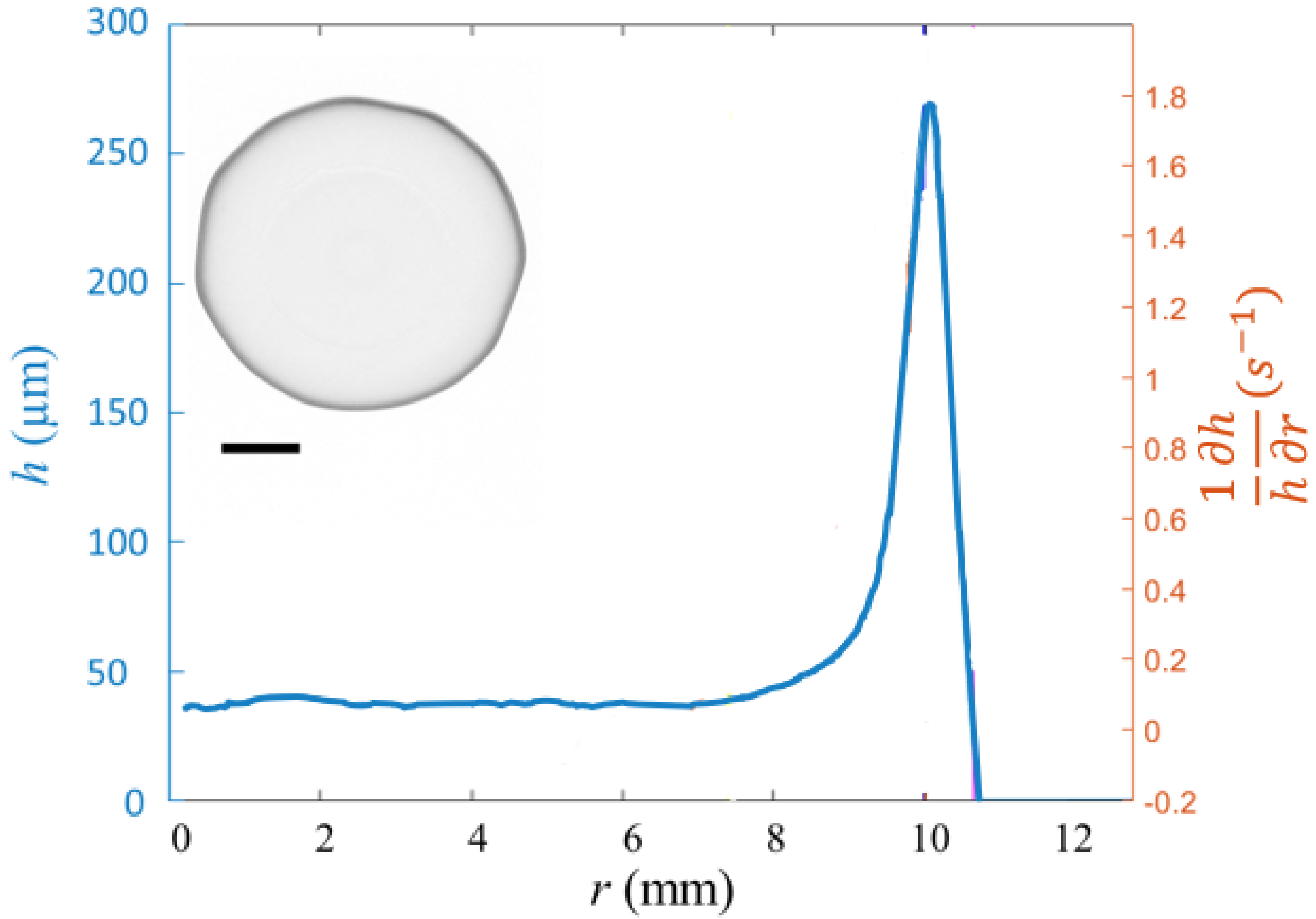
$\eta = 365$ mPas



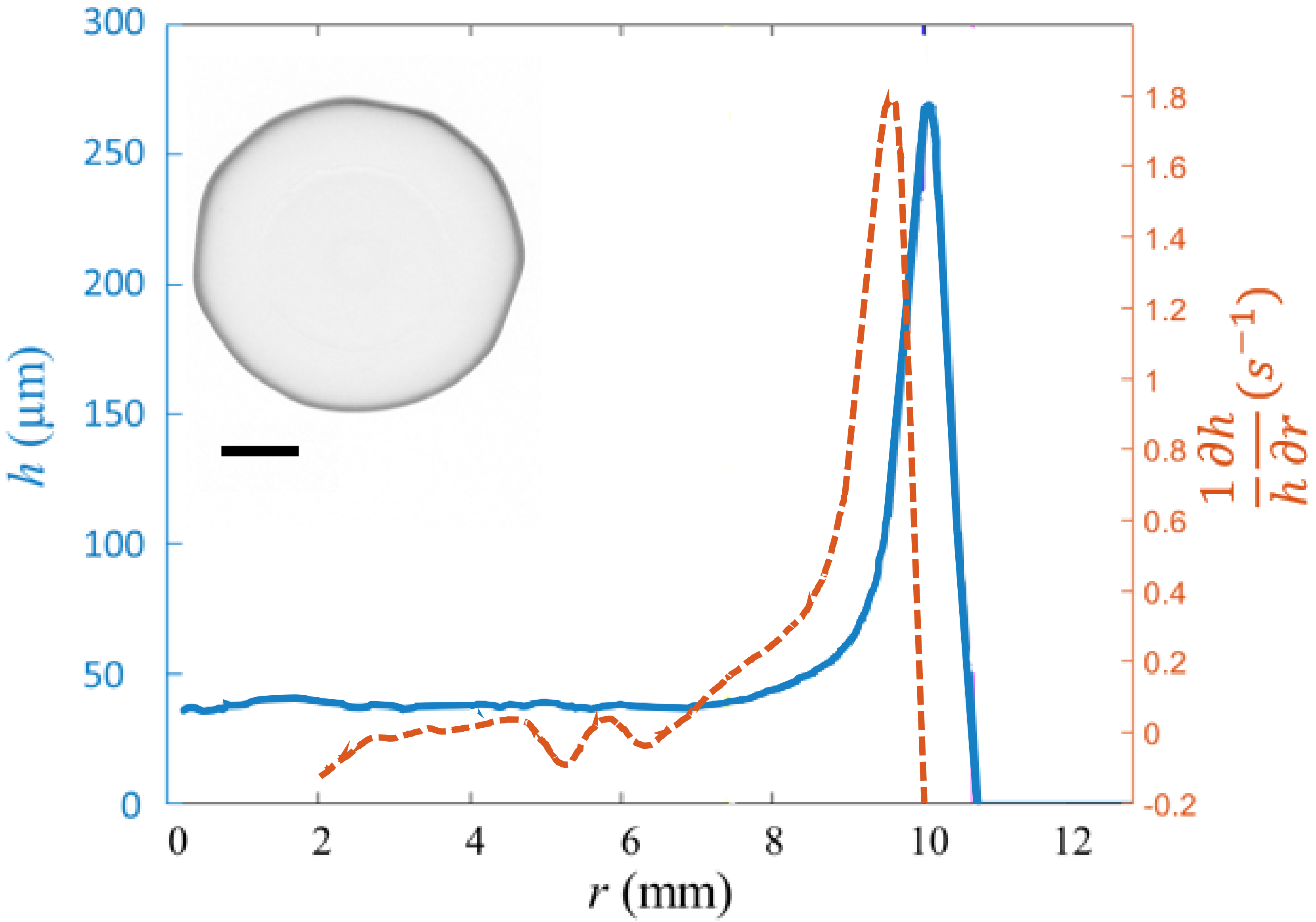
$\eta = 813$ mPas



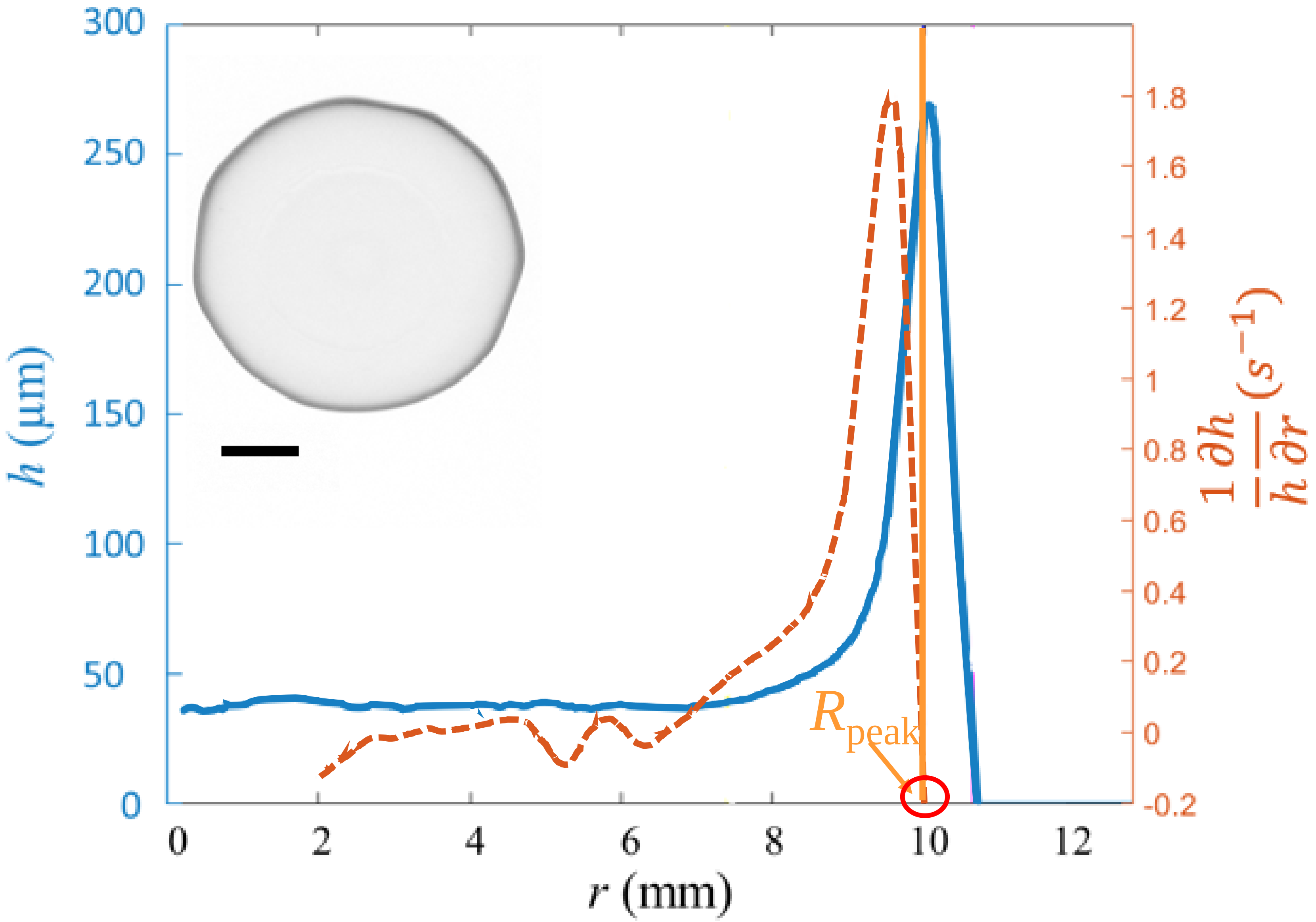
Measurement of the rim



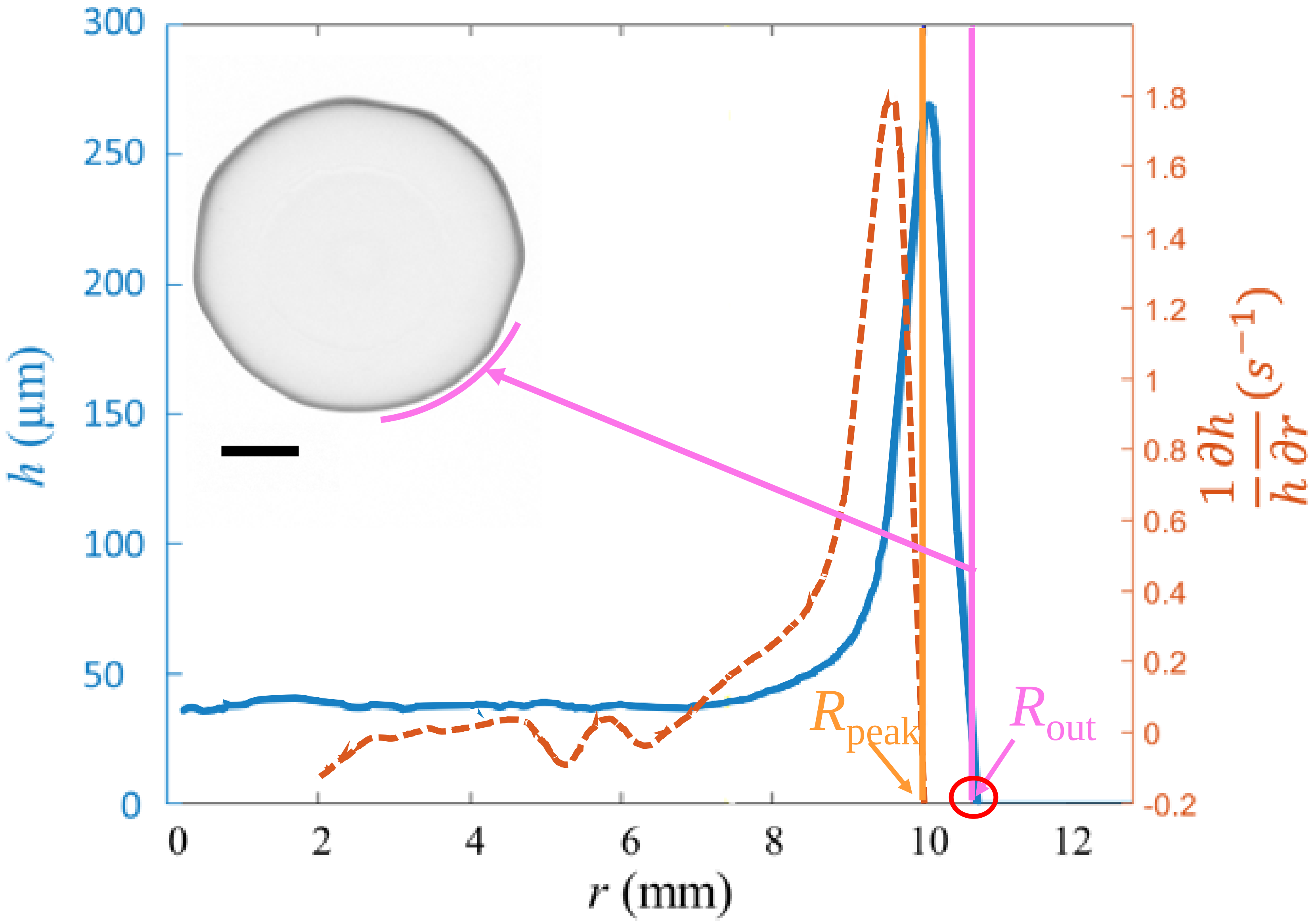
Measurement of the rim



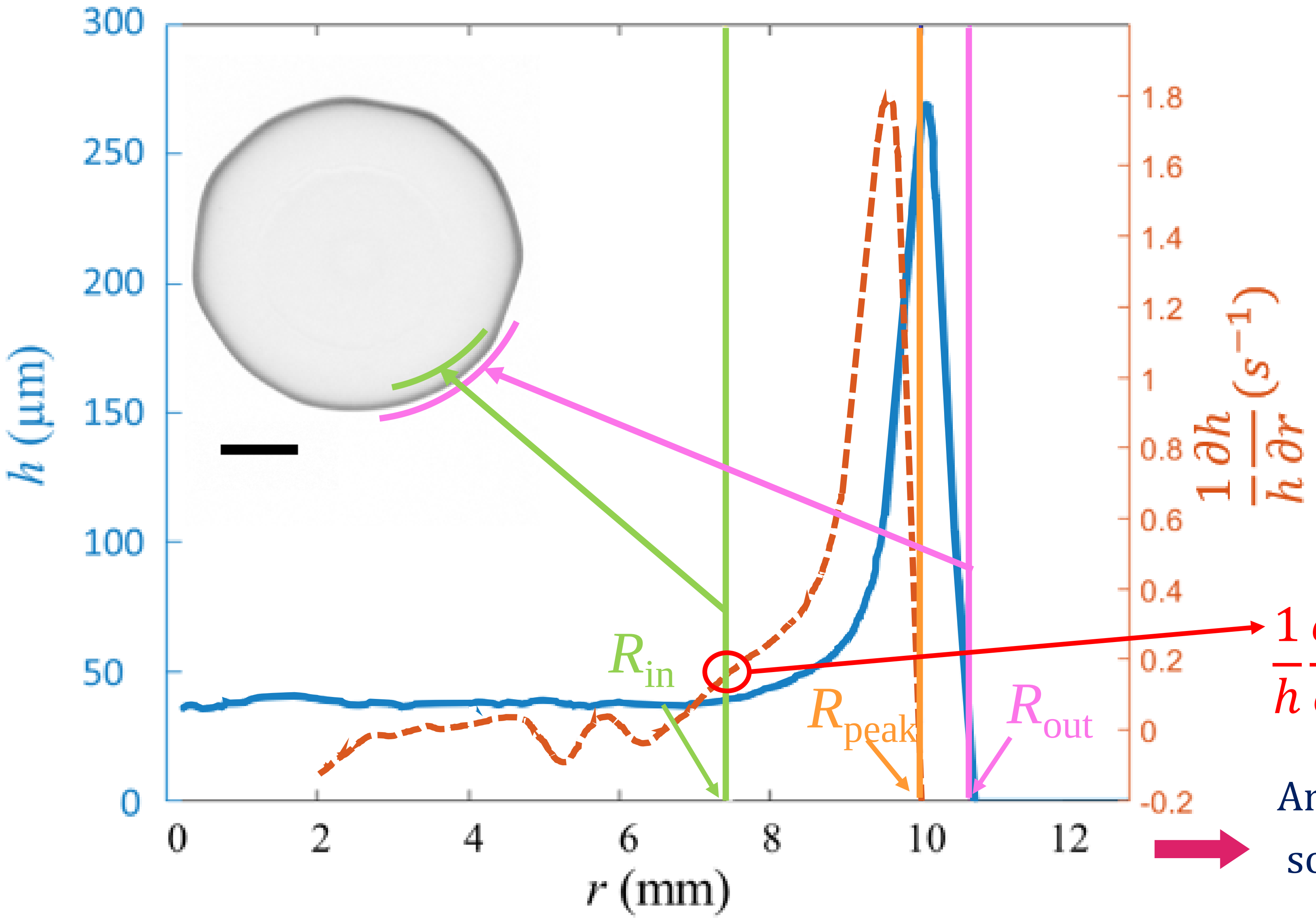
Measurement of the rim



Measurement of the rim



Measurement of the rim



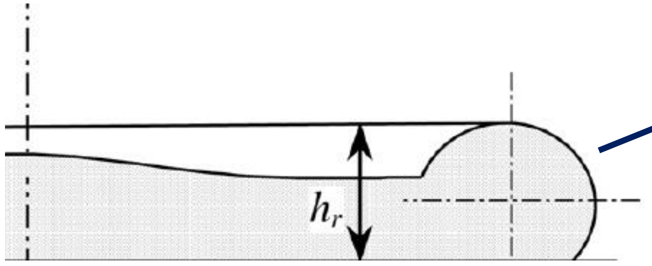
$$\frac{1}{h}\frac{\partial h}{\partial r} = 0.15\ s^{-1}$$

Arbitrary criterion to evaluate the rim

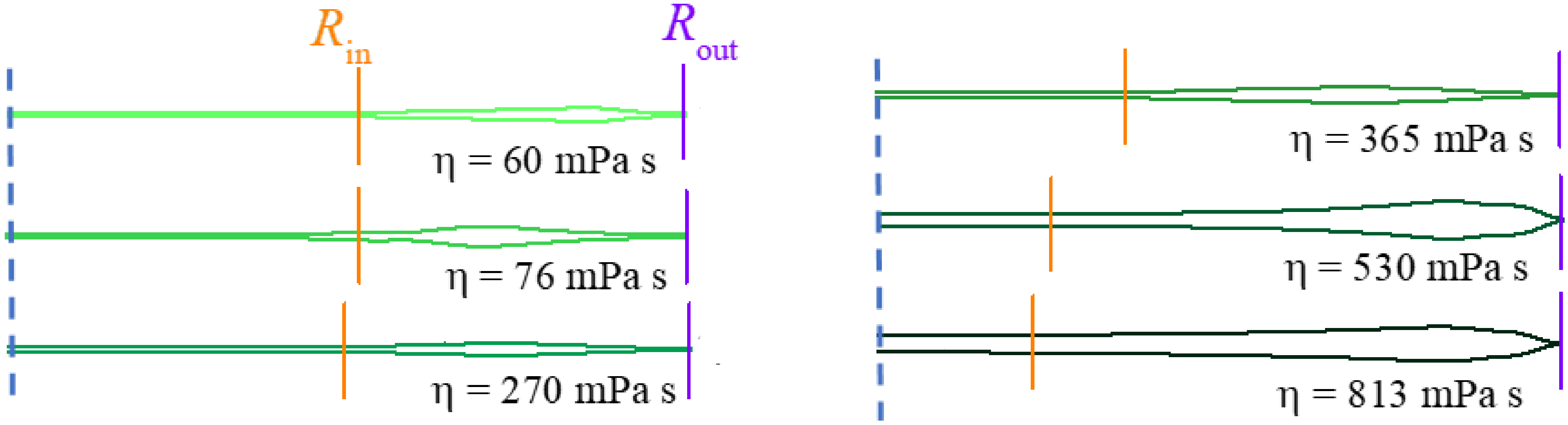
so, we also use $\frac{1}{h}\frac{\partial h}{\partial r}$ 0.10 and 0.20 s^{-1}

for robustness

Shape of the rim



Usually assumed shape



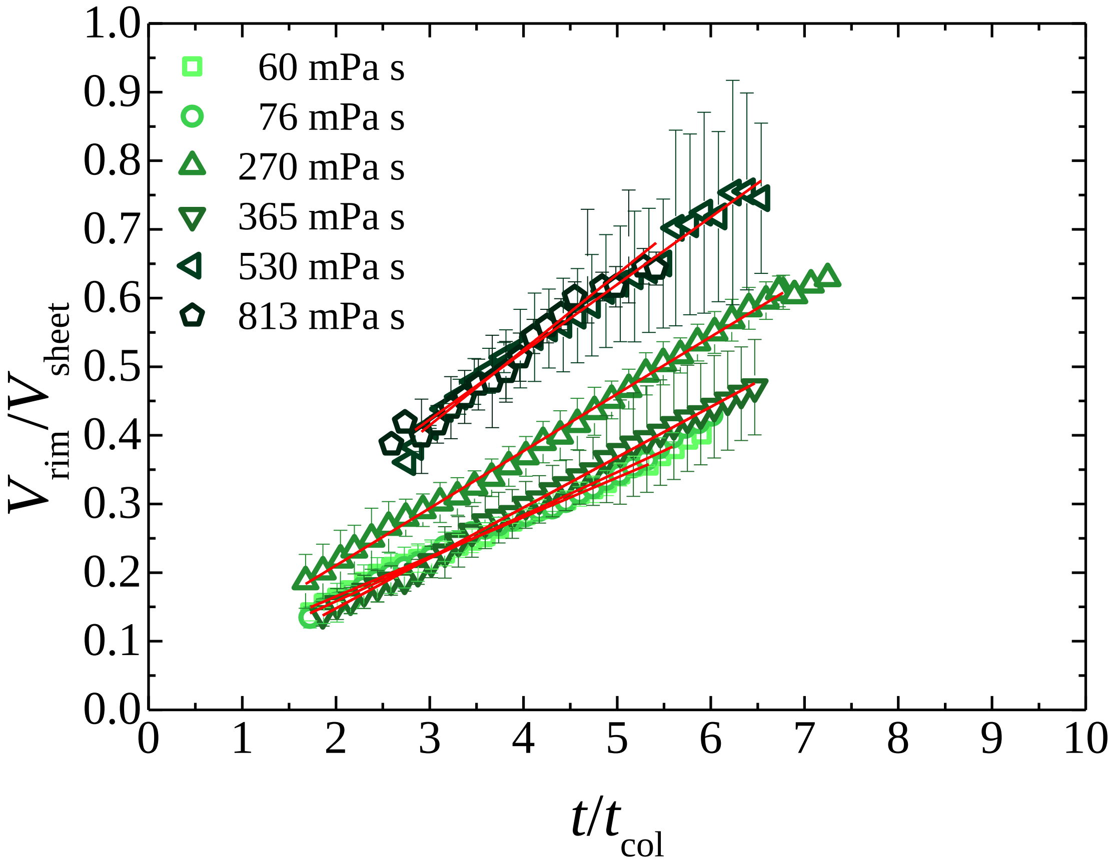
The shape is not circular

Volume of the rim

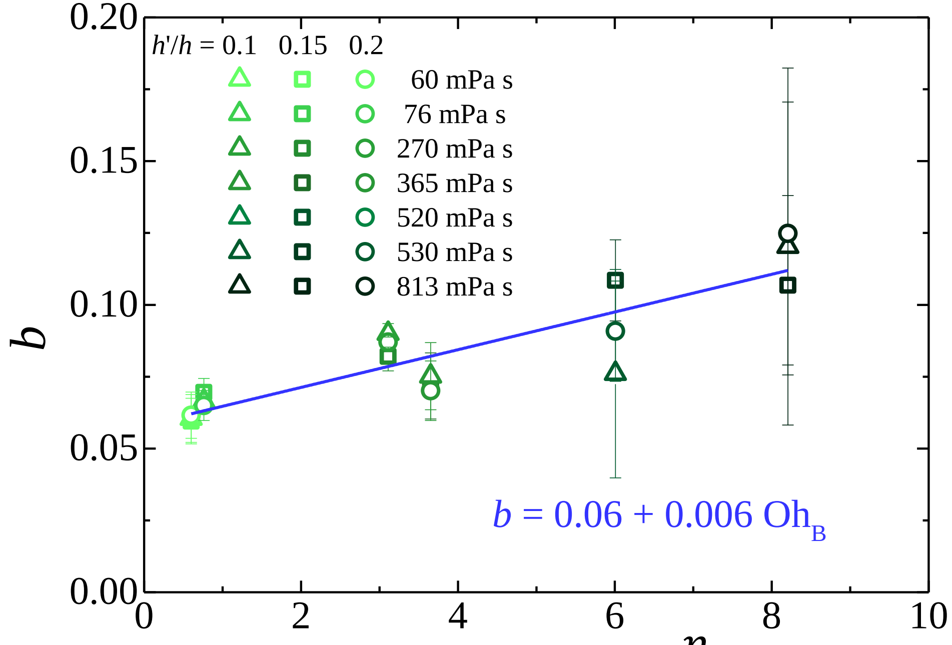
$$V_{rim}(t) = \int_{R_{in}}^{R_{out}} 2\pi r(t)h(r,t)dr$$

- The rim appears later for high viscosity fluids.
- The relative volume of the rim increases linearly with time.
- The non dimensional filling rate, *b*, of the rim is constant for each sample and weakly increases with sample viscosity.

$$b = \frac{1}{V_{sheet}}\frac{dV_{rim}}{dt}t_{col} = C_1 + C_2 Oh_B$$

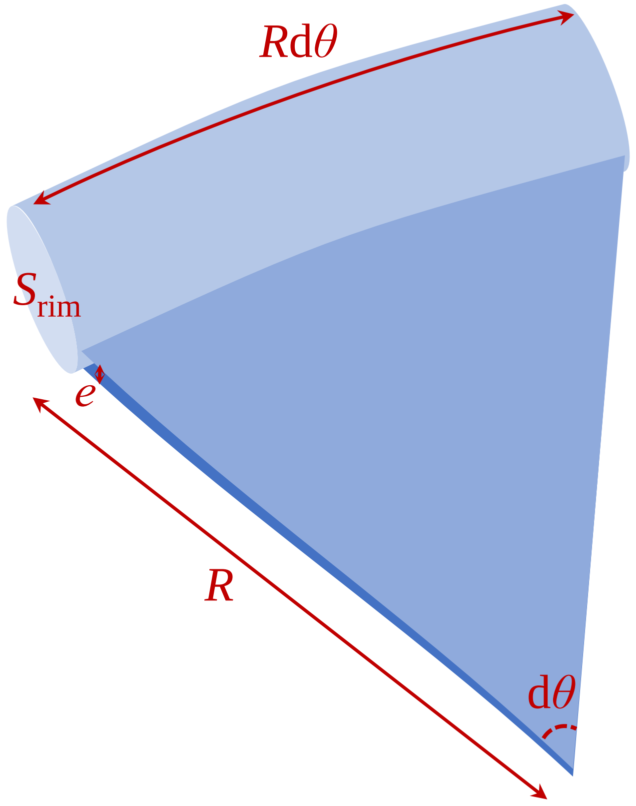


$t_{col} = d_0/v_0$



$$Oh_B = \frac{\eta_B}{\sqrt{\rho\gamma d_0}}$$

Filling velocity



Momentum balance for a portion of the rim

$$\frac{d(m\dot{R})}{dt} = \dot{m}u - 2\gamma R d\theta + 2Rd\theta e\eta \frac{\dot{R}}{R}$$

Rate of change of the rim momentum

Inertial forces

Surface tension

Viscous forces

m = Time varying mass of the portion of rim

u = Eulerian radial velocity of the liquid

e = Thickness at the entrance of the rim

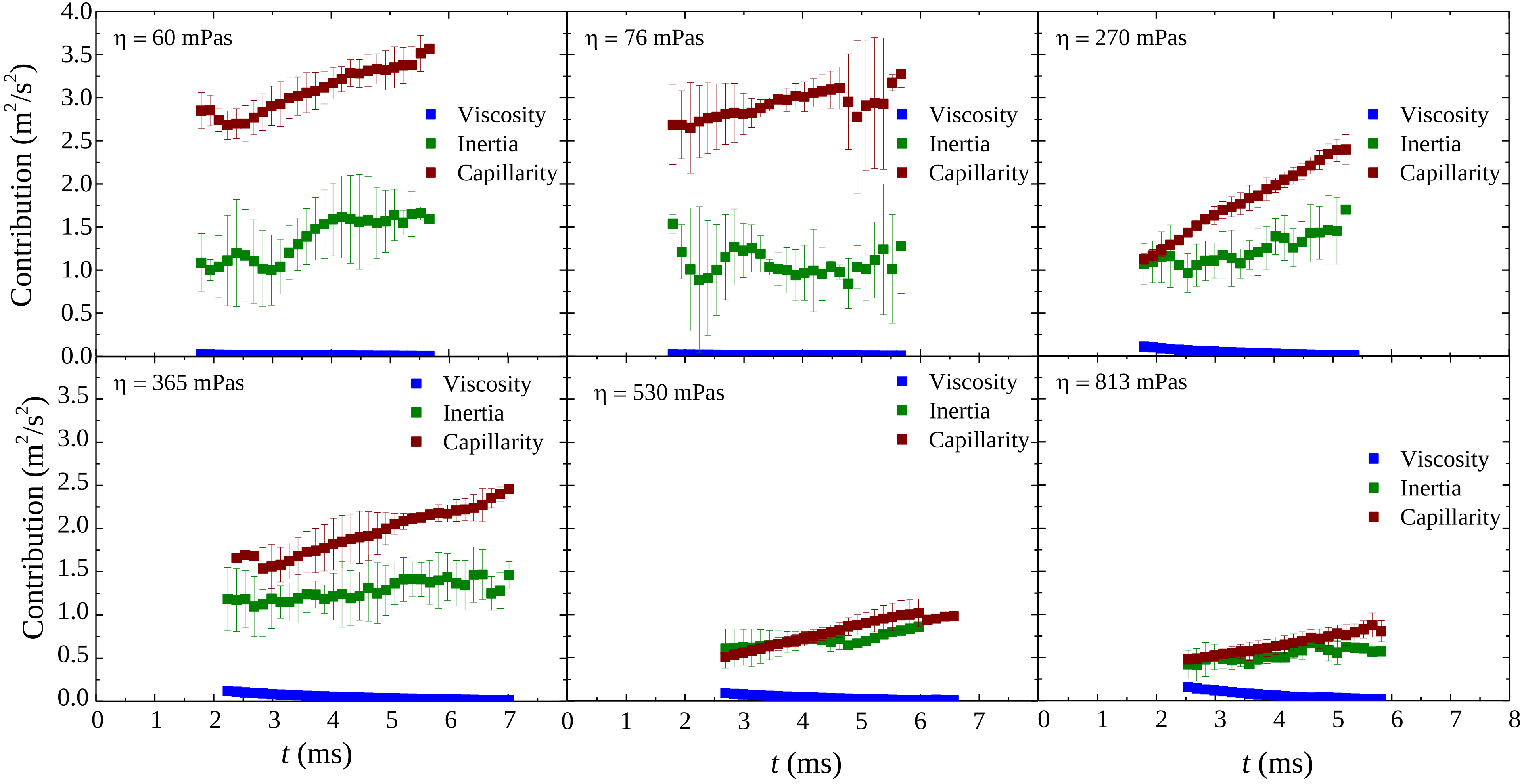
$$v_{fill} = \sqrt{\frac{2\gamma}{e\rho} - \frac{2\eta}{\rho}\frac{\dot{R}}{R} + \frac{V_{rim}}{2e\pi}\frac{\ddot{R}}{R}}$$

$$v_{fill} = u - \dot{R}$$

Contributions from:

- Capillarity
- Viscosity
- Inertia

Contributions to filling velocity



Capillarity = $\left|\frac{2\gamma}{e\rho}\right|$

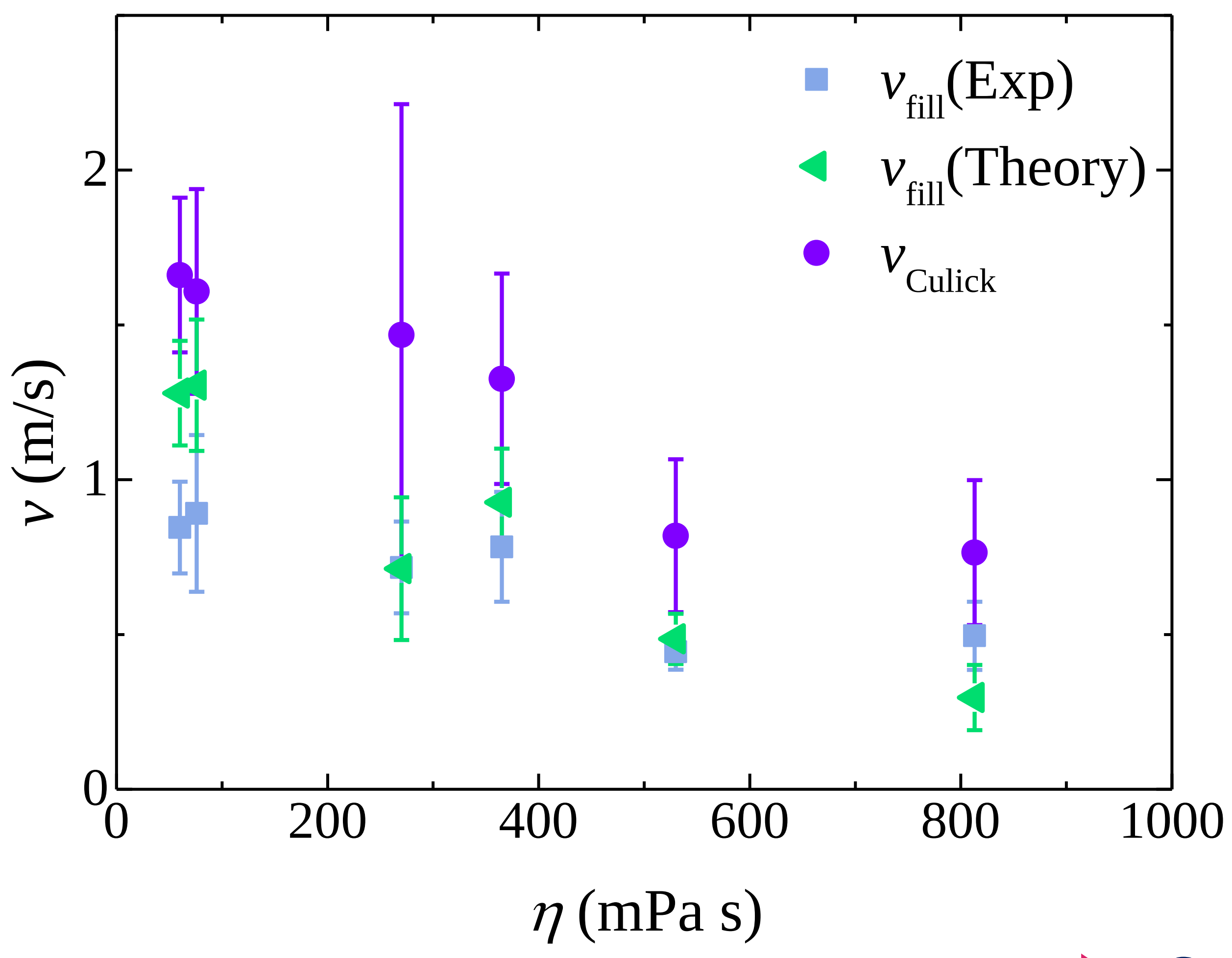
Inertia = $\left|-\frac{2\eta}{\rho}\frac{\dot{R}}{R}\right|$

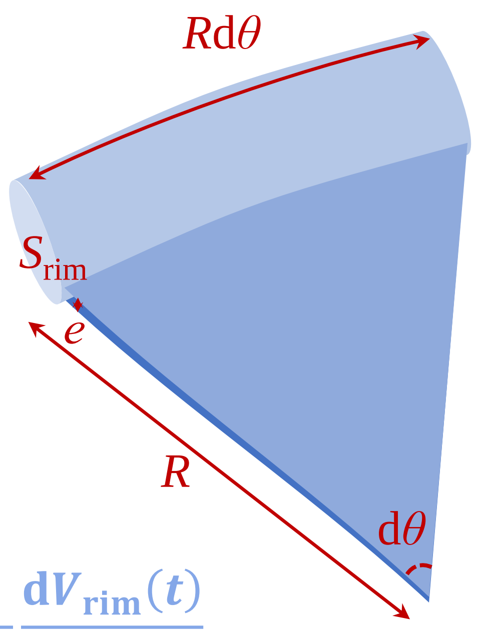
Viscosity = $\left|\frac{V_{rim}}{2e\pi}\frac{\ddot{R}}{R}\right|$

➡ Viscous contribution negligible

➡ Inertia and surface tension comparable

Filling velocity





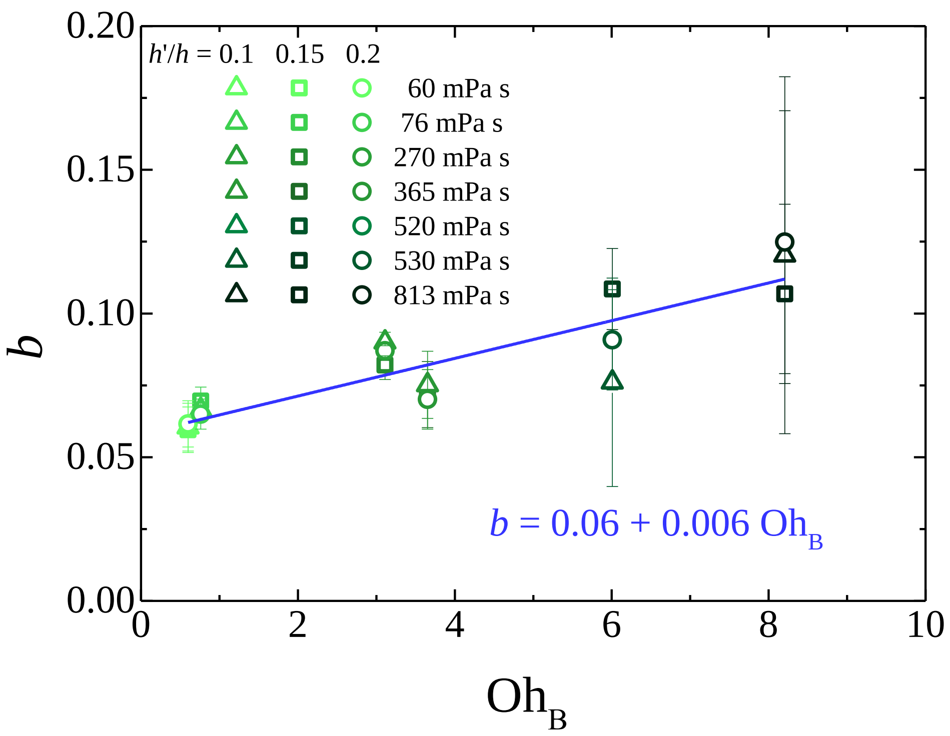
$$v_{\text{fill}}(\text{Exp}) = \frac{1}{2e(t)\pi R(t)} \frac{dV_{\text{rim}}(t)}{dt}$$

$$v_{\text{fill}}(\text{Theory}) = \sqrt{\frac{2\gamma}{e\rho} - \frac{2\eta}{\rho}\frac{\dot{R}}{R} + \frac{V_{\text{rim}}}{2e\pi}\frac{\ddot{R}}{R}}$$

$$v_{\text{Culick}} = \sqrt{\frac{2\gamma}{e\rho}}$$ (Typically used for the rim)

Good theoretical prediction of the filling velocity < v_{Culick}

Filling rate



$$b = \frac{1}{V_{sheet}}\frac{dV_{rim}}{dt}t_{col} = 0.06 + 0.006\ Oh_B$$

$$v_{\text{fill}}(\text{Exp}) = \frac{1}{2e(t)\pi R(t)}\frac{\text{d}V_{\text{rim}}(t)}{\text{dt}}$$

$$\frac{1}{V_{sheet}}\frac{\text{d}V_{\text{rim}}(t_{max})}{\text{dt}} \approx \frac{2v_{fill}}{R_{max}}$$

$$R_{max}(\eta) = R_{max,\eta(\to 0)}\sqrt{1 - 0.025\ \text{Oh}_B} \quad (1)$$

$$\frac{1}{V_{sheet}}\frac{\text{d}V_{\text{rim}}(t_{max})}{\text{dt}}t_{col} \approx \frac{2v_{fill}t_{col}}{R_{max,\eta(\to 0)}} + \frac{v_{fill}t_{col}}{R_{max,\eta(\to 0)}}0.025\ \text{Oh}_B$$

$$\frac{1}{V_{sheet}}\frac{dV_{rim}}{dt}t_{col} \approx 0.10 + 0.001\ Oh_B$$

Same order of magnitude

[(1)]Louhichi, A., Charles, C-A, Phou, T., Vlassopoulos, D., Ramos, L. & Ligoure, C., 2020, Phys. Rev. Fluids 5, 053602

Time for the apparition of the rim

Before the rim appears:

$$v_{\text{fill}} = \sqrt{\frac{2\gamma}{e\rho} - \frac{2\eta}{\rho}\frac{\dot{R}}{R} + \cancel{\frac{V_{\text{rim}}}{2e\pi}\frac{\ddot{R}}{R}}}$$

$$v_{\text{fill}} = v_{Culick}\sqrt{1 - \dot{\varepsilon}t_\eta}$$

Culick velocity

$$v_{\text{Culick}} = \sqrt{\frac{2\gamma}{e\rho}}$$

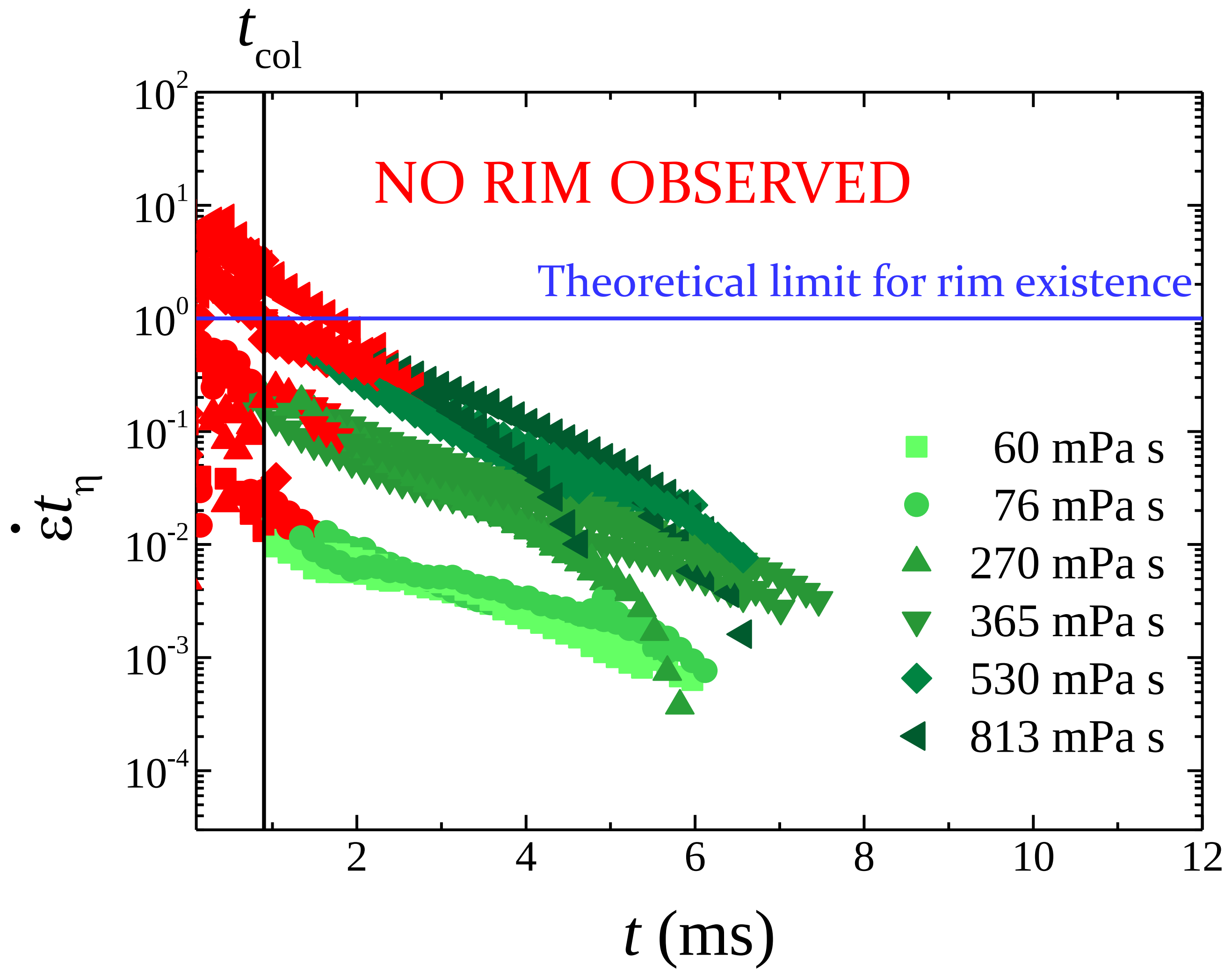
Extensional rate

$$\dot{\varepsilon} = \frac{\dot{R}}{R}$$

Viscous time

$$t_\eta = \frac{\eta e}{\gamma}$$

$$\boxed{\text{If} \quad \dot{\varepsilon}t_\eta > 1 \quad \text{NO RIM!}}$$



Conclusions

- ✓ Measure the thickness field of expanding sheets of Newtonian fluids on a repellent surface and quantitatively characterize the rim bounding the sheet
- ✓ Provide direct observation of the shape of the rim bounding an expanding sheet with a cross section very far from the circular one usually assumed.
- ✓ The filling rate and filling velocity of the rim are measured experimentally and rationalized theoretically
- ✓ Filling velocity is significantly lower than Culick's velocity (generally used to describe the growth of a rim bordering a free liquid sheet) due to the non negligible effect of the inertial forces
- ✓ Viscous effects negligible after the emergence of the rim but play an important role in the time at which the rim will appear

Aknowledgement

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Ameur Louhichi

INITIAL TRAINING NETWORK

Thank you for your attention